WHEN INTEREST RATES GO LOW, SHOULD PUBLIC DEBT GO HIGH?

Johannes Brumm
Xiangyu Feng
Laurence J. Kotlikoff
Felix Kubler

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ABSTRACT

Is deficit finance, explicit or implicit, free when borrowing rates are routinely lower than growth rates? Specifically, can the government make all generations better off by perpetually taking from the young and giving to the old? We study this question in simple closed and open economies and show that achieving Pareto gains requires implausible calibrations. Even then, the gains reflect, depending on the economy’s openness, improved intergenerational risk-sharing, improved international risk-sharing, and beggaring thy neighbor – not intergenerational redistribution per se. Low government borrowing rates, including borrowing rates running far below growth rates, justify improved risk-sharing between generations and countries. They provide no convincing basis for using deficit finance to redistribute from young and future generations or other countries.

Johannes Brumm
Karlsruhe Institute of Technology
Waldhornstr. 27
76131 Karlsruhe
Germany
jo.brumm@gmail.com

Xiangyu Feng
Boston University
270 Bay State Road
Boston, MA 02215
xyfeng@xmu.edu.cn

Laurence J. Kotlikoff
Department of Economics
Boston University
270 Bay State Road
Boston, MA 02215
and NBER
kotlikoff@gmail.com

Felix Kubler
University of Zurich
Plattenstrasse 32
CH-8032 Zurich
Switzerland
and Swiss Financial Institute
fkubler@gmail.com
1 Introduction

In his thought-provoking article Blanchard (2019), Olivier Blanchard suggests that implicit and explicit intergenerational redistribution – henceforth, deficit finance – may come at no cost to future generations given the average historical gap between the U.S. growth rate and the real interest rate on U.S. Treasuries. His analysis is framed within a stochastic, two-period overlapping-generations (OLG) economy. We revisit Blanchard’s simple closed-economy model and also examine two open-economy variants with three questions in mind. First, can deficit finance effect Pareto improvements in the ex-ante (pre-policy) expected utility levels of current and future generations? Second, do potential closed-economy Pareto gains reflect intergenerational redistribution per se or simply improved intergenerational risk-sharing? Third, do potential domestic Pareto gains in open economies reflect domestic intergenerational redistribution or simply improved intergenerational and international risk-sharing as well as beggaring thy neighbor’s capital stock?

To preview our findings, deficit finance, which we implement via a defined-benefit, pay-go, social security system (henceforth DB) can Pareto-improve Blanchard’s economy, but only under, in our view, implausible assumptions. Moreover, when such domestic efficiency gains arise, improved risk-sharing, beggar-thy-neighbor effects, or both are the reason. Stated differently, when deficit finance Pareto improves ex-ante expected utility (henceforth EAU) of current and future domestic residents, it does so because it improves risk-sharing, free rides on global capital formation, or both. Indeed, in Blanchard’s closed-economy model, if one first implements an efficient risk-sharing policy – in which the young give to the old in some states and receive from the old in other states – adding deficit finance to the policy mix serves only to redistribute, helping early generations at the price of hurting future generations. In the open economy, improved risk-sharing, including improved international risk-sharing, not deficit finance, is the path to making all generations, including foreign generations, better off.

Like Blanchard (2019), we assume Epstein and Zin (1989) preferences, but apply these preferences consistently in deriving EAU. Doing so entails, in contrast to Blanchard’s approach, evaluating uncertainty over the state in which an agent is born with the same degree of risk aversion with which the agent evaluates the uncertainty of old-age consumption. This neither influences agents’ behavior, nor the model’s calibration. But it does provide greater scope for Pareto improvements via deficit finance, which makes our results – the inability to Pareto improve under realistic calibrations and uncovering the underlying sources of Pareto gains (when they arise) – even more striking.

We begin with Blanchard’s closed economy and then add a second country, considering both a stylized symmetric case and one in which the two countries represent the U.S. and the rest of the world (RoW). In the closed economy, we convey the importance of risk-sharing to the success of DB pay-go in four ways. First, we decompose a generation’s EAU gains into three components – a risk-neutral effect, a life-cycle-risk effect, and a cohort-risk effect. The risk-neutral effect (RNE) isolates the EAU impact of policy-induced changes in average, realized, cohort-specific consumption levels. The life-cycle-risk effect (LRE) captures the EAU impact
of old-age consumption risk given an agent’s state of birth. And the cohort-risk effect (CRE) captures the EAU impact of agents not knowing the state into which they will be born. Our decomposition relates the percent change in EAU to the sum of RNE, LRE, and CRE. As we show, increases in EAU of those alive in the long run (henceforth, long-run EAU) are fully explained by LRE and CRE, which capture improved risk-sharing both post- and pre-birth.¹

Second, we show that running deficits of the same average size, but with less inherent risk-sharing can vitiate what we view as the slender prospects for a Pareto improvement. Third, we consider a two-way transfer scheme where the old transfer to the young when shocks are good and the young transfer to the old when shocks are bad – with the policy redistributing nothing, on average, across generations. We show that such a scheme a) is Pareto improving when DB pay-go is not, b) Pareto dominates DB pay-go even when such policy is Pareto improving, and, once implemented, c) exhausts the potential for additional Pareto gains via deficit finance. Fourth, we show that Pareto gains to EAU, in the special cases in which they arise, disappear in the absence of Blanchard (2019)’s assumption that the young receive a safe endowment in addition to their risky wages.²

Adding an equally large second country expands somewhat the scope for domestic efficiency gains. I.e., generating higher EAU levels for domestic residents is feasible for a slightly larger range of parameters than in the closed economy. However, this home-country improvement typically reflects beggaring thy neighbor, specifically the ability of the home country to consume not just some of its own seed corn, but also some of the seed corn that would otherwise be planted (invested) abroad. With sufficiently low risk-free rates as well as exceedingly small risk premia, both countries can gain from domestic deficits. But what’s good for the goose is good for its symmetric gander. Since the home country gains more than the foreign country, the foreign country will, surely, engage in its own deficit finance. The Nash equilibrium to the game of free riding on the global supply of capital entails each country setting its own intergenerational redistribution above the cooperative outcome. This leaves both countries worse off for their effort.

Our third model takes the U.S. as the home country and the RoW as the foreign country. The model includes compensated foreign-investment taxes to achieve a realistic pattern of cross-country asset holdings.³ Surprisingly, in this more realistic setting, the domestic EAU effects of DB pay-go are only slightly more favorable than in the closed economy. The reduction in U.S. saving and, thus, global capital formation does produce a beggar-thy-neighbor effect. Indeed, the U.S. experiences only about one quarter of the reduction in domestic capital that would arise were the economy closed. Stated differently, the RoW takes most of the hit to global investment. But given our assumed incomplete, international financial markets, domestic DB pay-go also benefits foreigners through improved international risk-sharing. Specifically, the reduction in

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¹Since increases in EAU decline monotonically as one moves from early to later generations, increases in long-run EAU ensures that all prior generations are also better off, i.e., it implies a Pareto improvement in the path of EAU.
²By analogy, Samuelson (1958)’s Pareto improvement from his Ponzi scheme disappears if one drops his no-storage assumption.
³Assuming real transaction costs of investing abroad would serve equally well.
saving in the home country reduces the global demand for the domestically-supplied safe bond, raising its return. This lets foreign agents build safer portfolios at lower cost.

To summarize, in the special cases in which they occur, the secret sauce underlying domestic Pareto-improving deficits in Blanchard’s model is improved risk-sharing, intergenerationally and internationally, plus beggaring thy neighbor. Appropriate bilateral risk-sharing policy can make all generations, foreign and domestic, better off. But beggaring thy children or thy neighbor is not an ingredient in the recipe. Indeed, as risk-sharing becomes more efficient, deficit finance, whether explicit or implicit, devolves to zero-sum generational and international games.

We proceed, in section 2, with a brief literature review. Section 3 presents the closed-economy model and motivates our EAU measure. Section 4 presents our closed-economy results and explains how we decomposes EAU changes into its risk neutral and risk-sharing components. Section 5 adds a second economy – initially identical to the first economy, but then treats the first economy as the U.S. and second economy as the rest of the world. Section 6 acknowledges that deficit finance may be justified in Keynesian and other settings even if isn’t justified in Blanchard (2019)’s. Section 7 summarizes and concludes.

2 Literature Review

Blanchard (2019) alights from a venerable literature, including Solow (1956), Samuelson (1958), Phelps (1961), Diamond (1965), and Tirole (1985), examining dynamic inefficiency in deterministic settings. His paper also connects to classic studies by Abel et al. (1989) and Zilcha (1990), which show that stochastic OLG economies that perpetually accumulate excess capital are dynamically inefficient – that is, with appropriate policy, all current and future agents can consume more in every state. After consulting the historical record, Abel et al. (1989) find no evidence that the U.S. or, indeed, any major OECD country, saves simply for the sake of saving. Yet, a stochastic OLG economy’s dynamic efficiency doesn’t ensure its Pareto efficiency – the inability to raise at least one current or future generation’s ex-ante utility without lowering that of any other.

Unfortunately, assessing Pareto efficiency is more challenging than evaluating dynamic efficiency. As Merton (1983), Ball and Mankiw (2007), Barbie et al. (2007), Bohn (1999), and others emphasize, stochastic OLG models are inherently Pareto inefficient. Current generations can’t share risk with future generations. Consequently, any government policy, including deficit finance, may generate a Pareto improvement merely because of its direct or indirect risk pooling. Merton (1983) provides an elegant demonstration of fiscal policy’s intergenerational risk-sharing capacity. He shows that a combination of consumption taxes, wage taxes, and old-age state pensions can fully share productivity, demographic, and factor-share shocks across generations. In a similar vein, Bohn (1999) demonstrates that investing social security’s trust fund in equities can better insure payment of future generations’ promised benefits.

Scheinkman and Weiss (1986) is an early paper showing how the introduction of a fundamentally worthless asset can improve risk-sharing in a model of infinitely lived agents. Whether
one calls this asset fiat money, government debt, or a state-contingent tax-transfer system, is immaterial. The key point, from our study’s perspective, is the longstanding recognition of the government’s ability to use fiscal arrangements as pseudo financial instruments that can Pareto improve. Also, as Samuelson (1958) and Tirole (1985)’s papers implicitly make clear, whether these policies are viewed (labeled) as government policy or as privately-initiated financial bubbles, makes no difference. Recent contributions focusing on the bubble component of government debt, with a particular focus on the low rates environment, include Jiang et al. (2019), Miao and Su (2021), Brunnermeier et al. (2020), and Reis (2021) – the later two emphasizing that growth rates in excess of safe rates can expand fiscal capacity, but only to a limited extend. Our admonition is that deficit finance’s economic justification rests on Pareto efficiency, not fiscal sustainability.

Hubbard and Judd (1987) and ˙Imrohoroglu et al. (1995) examine the state’s ability to share micro, not macro risk, particularly idiosyncratic longevity risk. They demonstrate that the risk-sharing of state pensions can raise long-run welfare despite the policy’s crowding out of capital. Their implicit message, which we reprise here, is that risk-sharing policy is distinct from and can be run independent of intergenerational redistribution policies. Krueger and Kubler (2006) explicitly distinguish social security’s roles in sharing risk and reducing capital formation. In their case, the risk arises from macro shocks that can drive wages and capital returns in different directions. Unlike Hubbard and Judd (1987) and ˙Imrohoroglu et al. (1995), they find that the long-run, average net welfare impact of pay-go social security is negative. Bovenberg and Uhlig (2008) also clarifies how the design of state pensions – precisely how deficit finance is structured – can impact intergenerational risk-sharing.

Hasanhodzic and Kotlikoff (2013) reach similar conclusions to Krueger and Kubler (2006), but do so in a larger scale (80-period) life-cycle model. Their study suggests that intergenerational risk, caused by macro shocks, is far smaller than suggested by models with fewer periods. Harenberg and Ludwig (2019) reach a different conclusion by combining correlated micro and macro shocks. The interaction of these shocks substantially exacerbates aggregate risk, making, in their model, risk mitigation more important than crowding out in determining the long-term gains from pay-go social security.

Ball and Mankiw (2007) and Barbie et al. (2007) provide general conditions for ex-ante and ex-interim (at a cohort’s time of birth) Pareto-efficient intergenerational risk-sharing. Ex-ante efficiency is, to repeat, our focus. Ex-interim efficiency, considered in Krueger and Kubler (2002), is a more stringent criterion for achieving Pareto improvements than that adopted here. It treats agents born in particular states as distinct and requires that all their policy-induced welfare changes be non-negative.

Blanchard (2019) ties the question of government debt and intergenerational transfers to the current low interest-rate environment. He, like Sergeyev and Mehrotra (2020) and Summers and Rachel (2019), argue that an increase of government debt may engender no fiscal costs. Sims (2019) critically examines this argument from a monetary perspective. Barro (2020) argues

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4If, as claimed here, risk-sharing is the sine qua non for Pareto-improving deficit finance, a surfeit of risk to share in realistically-timed models raises further doubt about the efficiency of intergenerational redistribution.
that understanding the consequences of low risk-free rates requires modeling the high equity premium, which he traces to rare disaster risk. Hellwig (2020) provides supporting theoretical arguments for the possibility that modern economies are dynamically inefficient. In particular, he shows that the presence of a long-lived productive asset is not inconsistent with dynamic inefficiency in the presence of transaction costs.

Blanchard et al. (2020) explore some implications of these arguments for EU fiscal rules. In an issue of the AEA papers and proceedings, Brumm et al. (2020), Evans (2020) and Hasanhodzic (2020) critically discuss some of the assumptions underlying Blanchard (2019). None of these papers consider either the importance of risk-sharing or beggar-thy-neighbor policy for assessing deficit policies. Nor do Ball and Mankiw (2021) who view low safe rates as reflecting market power and find that deficit finance may reduce welfare even with very low safe rates. Since risk and market power are arguably the major drivers of the wedge between safe rates and risky rates, their findings complement ours in providing a word of caution about taking low interest rates as a reason to raise public debt.

3 Blanchard’s Closed Economy Revisited

Here we revisit Blanchard (2019)’s closed economy and its calibration. But we replace Blanchard’s welfare measure with one that fully accords with Blanchard’s assumed Epstein and Zin (1989) (EZ) preferences. Doing so expands the set of parameter values for which deficit finance is Pareto improving. Even so, this set is quite narrow, comprising very low risk-free rates and/or risk-premia, as we show in Section 4.

3.1 OLG Economy With Intergenerational Transfers

Blanchard (2019)’s OLG model is bare bones. Agents live for two periods, consuming $c_{y,t}$ when young and $c_{o,t}$ when old. Their utility is EZ (homothetic Kreps-Porteus), with an intertemporal elasticity of substitution (IES) of 1, a risk-aversion parameter denoted $\gamma$, and a discount rate of $\beta = \beta/(1 - \beta)$. Blanchard (2019) specifies these preferences with the utility function

$$(1 - \beta) \log c_{y,t} + \frac{\beta}{1 - \gamma} \log E_t [c_{o,t+1}^{1-\gamma}] . \quad (1)$$

By a monotone transformation via the exponential function we get:

$$c_{y,t}^{(1 - \beta)} E_t \{c_{o,t+1}^{1-\gamma}\}^{\frac{\beta}{1 - \gamma}} . \quad (2)$$

This is in line with Epstein and Zin (1989)’s original formulation. It also has the advantage of being homogeneous of degree one so that variations in utility corresponds to variations in consumption, i.e., a given percentage increase in consumption in all states produces the same percentage increase in utility.

Output is Cobb-Douglas with total factor productivity, $A_t$, fluctuating randomly around a
fixed mean. When young, agents work, earn the wage \( W_t \), receive a safe endowment \( E \), and pay a net tax \( T_t \). When old, agents receive a net transfer \( T_{t+1} \). Note, in each period, transfers to the old equal taxes on the young. There are two assets – capital, whose return is risky and whose principal depreciates fully each period, and a risk-free bond in zero aggregate supply. Denote the return on capital by \( R_t \) and the risk-free return on the bond by \( R_{ft} \). The generation born at \( t \) solves

\[
\max_{c_{y,t},c_{o,t+1},k_{t+1},b_{t+1}} c_{y,t}^{(1-\beta)} E_t \left\{ c_{o,t+1}^{1-\gamma} \right\}^{\frac{1}{\gamma}} \quad \text{s.t.} \quad \begin{align*}
    c_{y,t} &= W_t + E - k_{t+1} - b_{t+1} - T_t \\
    c_{o,t+1} &= k_{t+1}R_{t+1} + b_{t+1}R^f_{t+1} + T_{t+1}.
\end{align*}
\]  

(3)

\( A_t, R_t, \) and \( W_t \), satisfy

\[
\log A_t = \epsilon_t, \quad \epsilon_t \sim_{i.i.d.} \mathcal{N} \{0, \sigma^2\}. 
\]  

(4)

\[
R_t = \alpha A_t k_t^{\alpha-1}. 
\]  

(5)

\[
W_t = (1-\alpha)A_t k_t^{\alpha}. 
\]  

(6)

The supply of capital is determined by the investment choices of the young. The net supply of bonds is (in the absence of government debt) zero. The risk-free rate is determined via market clearing.

Below, we contrast the laissez-faire economy, \( T_t = 0 \), with a tax-transfer policy that is constant over time, \( T_t = T > 0 \). We call this policy defined-benefit (DB) pay-go. While we follow Blanchard (2019) in focusing on this transfer scheme, we also consider other schemes – defined contribution, constant debt, and two-way transfers (see Sections 4.3, 4.4, and 5.2).

### 3.2 Calibration

Following Blanchard (2019)’s, we set capital’s share, \( \alpha \), at 0.33, the standard deviation of the productivity shock, \( \sigma \), at = 0.2, and the fixed endowment, \( E \), at one half average wages in the no-policy stochastic steady state.\(^5\) Also, following Blanchard (2019), we consider, except in Section 4.5, only cases of \( T \leq E \), which ensures feasibility.\(^6\)

To calibrate preference parameters, we first fix a pair of targets for the unconditional mean of the risk-free rate (RFR), \( E_0 \left[ R^f_t \right] \), and the risk premium (RP), \( E_0 \left[ R_t - R^f_t \right] \). We then choose pairs of \( \beta \) and \( \gamma \) that meet the targets.\(^7\) We focus on two cases. Baseline 1 has an annualized risk-free rate of -1 percent and an annualized risk premium of 3 percent. To hit these targets in the closed economy, we set \( \gamma \) to 19.0 and annualized \( \beta \) to 0.933. Baseline 2 features an even

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\(^5\)In our two baseline cases, \( E \) represents about 40 percent of average total output, the later being production plus the endowment. The endowment assumption thus effectively reduces capital share of total output to 20 percent.

\(^6\)For \( T \geq E \), there will be cases of game over – realizations of \( A_t \) in which the young have too little resources to cover their pay-go contributions. This “game over” limit, examined by Evans et al. (2012), plays a key role in Tirole (1985) and other studies of bubbles of finite value.

\(^7\)When there are two countries, both are assumed to have the same preferences.
lower risk-free rate of -2 percent and a risk premium of 3. Here $\gamma$ is set at 19.2 and $\beta$ at 0.944. The values of $\gamma$ and $\beta$ needed to hit these two targets are quite sensitive to TFP risk $-\sigma$, which Blanchard sets at 0.2. Fortunately, as shown in Appendix B, our main results aren’t particularly sensitive to changes in $\sigma$ and the associated changes in $\gamma$ and $\beta$.

Blanchard’s calibration abstracts from both population and TFP growth. The average postwar U.S. population growth rate was roughly 1 percent and the average growth rate of TFP was around 1.5 percent. Hence, a -2 percent differential between the average safe rate and the average growth rate corresponds to an annual safe rate of about 0.5 percent in a model where population growth and TFP growth are matched to historical averages.\(^8\) The historical average real return on the 1-year U.S. Treasury bill rate is 0.6 percent. Hence, calibrating, as we do, a real risk-free rate/growth rate differential of -2 percent and -1 percent (net of growth) in our two baselines appears to capture the range of empirically plausible parameters.

What about the risk premium? The historical average risk premium on equity has been well above 4 percent. On the other hand, returns to physical capital as measured from national product accounts seem to lie slightly below 4 percent. Of course, physical capital is just a portion of U.S. national wealth, whose real return has averaged 6.5 percent in the postwar era. It averaged 9.5 percent between 2010 and 2019.\(^9\) In sum, our baseline assumption of a 3 percentage point (pp) risk premium seems at the low end of what’s empirically reasonable. Nonetheless, we adopt this value to give deficit finance the benefit of the doubt. For, as we and Blanchard (2019) show, adopting a higher and, to us, more plausible risk premium rules out Pareto-improving deficit (DB pay-go) policy.

These crucial calibration targets complete our description of the closed-economy’s calibration. For the open-economy cases, most of the calibration details carry over. Remaining details are described in section 5 and the Appendix C.

### 3.3 Computation

We first describe our computational approach for the closed economy model and then turn to the two-country case. The state of the closed economy at time $t$ is characterized by capital, $k_t$, accumulated in the previous period and TFP, $A_t$, determined exogenously. These variables jointly determine factor prices and consumption of the old,

\[
R_t = \alpha A_t k_t^{1-\alpha} \\
W_t = (1-\alpha)A_t k_t^\alpha \\
c_{o,t} = k_t R_t + T_t.
\]

\(^8\)This said, U.S. population growth is far from stationary and is projected to decline to zero in the second half of this century. See Aksoy et al. (2019)). For its part, TFP growth has slowed in this Century. Whether this reflects mis-measurement, a temporary decline, or a new normal (see, e.g., Crafts (2018)) remains to be seen.

\(^9\)Authors calculations based on NIPA data and the Federal Reserve’s Financial Accounts.
Given the state and prices in $t$, current choices of the young and the risk-free rate, $(c_{y,t}, k_{t+1}, R^f_{t+1})$, satisfy the following system of equilibrium conditions – two Euler equations of the young, and their budget constraint:

\[
\frac{1 - \beta}{c_{y,t}} = \beta \frac{E_t \{ R_{t+1} c_{o,t+1}^{\gamma} \}}{E_t \{ c_{o,t+1}^{1-\gamma} \}},
\]
\[
\frac{1 - \beta}{c_{y,t}} = \beta \frac{R^f_{t+1} E_t \{ c_{o,t+1}^{\gamma} \}}{E_t \{ c_{o,t+1}^{1-\gamma} \}},
\]
\[
c_{y,t} = W_t + E - k_{t+1} - T_t,
\]

where the risky return, $R_{t+1}$, and consumption when old, $c_{o,t+1}$, depend, as in equation (7), on capital saved for next period, $k_{t+1}$, and on next period’s realization of TFP, $A_{t+1}$:

\[
\log A_{t+1} = \epsilon_t, \quad \epsilon_t \sim_{i.i.d.} \mathcal{N} \{0, \sigma^2\}.
\]

We solve our closed- and open-economy models on a period-by-period basis for 1000 periods, drawing shocks along the path. When the economy is closed, the solution devolves to solving, for each period, three equations in three unknowns – the risk free rate, the consumption of the young, and the saving of the young. In the open economy, there are seven equations in seven unknowns – the risk free rate, the consumption and saving of the young in each economy, home-country holdings of foreign capital, and foreign-country holdings of domestic capital. We find exact solutions to the relevant equations using a non-linear solver and determining expected values via Gauss-Hermite quadrature of order 20.

### 3.4 Ex-Ante Utility

Pay-go policies redistribute to the initial elderly. Hence, the crucial question is how such policies affect future generations. We assess their welfare using the following ex-ante expected utility function, which is homogeneous of degree one in consumption. This specification incorporates our monotone transformation, given in (2), of Blanchard’s utility function, given in (1).

\[
U^t_0 = \left( E_0 \left[ (1 - \beta) \log c_{y,t} + \frac{\beta}{1 - \gamma} \log E_t \{ c_{o,t+1}^{1-\gamma} \} \right] \right)^{\frac{1}{1 - \gamma}},
\]

where 0 is the time of assessment, i.e., when a policy choice is made, and $t > 0$ is the time of birth. This measure evaluates uncertainty about the state in which an agent is born with the same degree of risk aversion with which the agent evaluates the uncertainty of old-age consumption. Blanchard (2019), in contrast, appears to evaluate long-run EAU via

\[
\tilde{U}^t_0 = E_0 \left[ (1 - \beta) \log c_{y,t} + \frac{\beta}{1 - \gamma} \log E_t \{ c_{o,t+1}^{1-\gamma} \} \right] = E_0 \left[ \log \left( (1 - \beta) E_t \{ c_{o,t+1}^{1-\gamma} \} \right)^{\frac{\beta}{1 - \gamma}} \right],
\]
which effectively assumes risk aversion of 1 with respect to the state in which an agent is born. That’s much lower than the risk aversion re old-age consumption needed to match the risk premium in the considered calibrations. Because of this difference, our measure (10) is more favorable to finding welfare improvements from deficit finance than is (11), as we illustrate in Appendix A.\footnote{But, to repeat, the choice between (10) and (11) has no impact on the calibration, as both specifications reflect the same preferences at the ex-interim stage when agents are alive and making choices.} In what follows, we motivate the use of EAU as defined in (10) in two ways – a simple example and a derivation from basic assumptions.

**Simple Example**

Consider the problem of evaluating, at time 0, the expected utility of an agent born at time 1 who lives for two periods and has Epstein-Zinn preferences as specified in (2). Suppose there are two equally likely states at time 1, A and B. Evaluated at time 1, the agent’s utility, conditional on state A, is

\[
U_1^A = c_1^{(1-\beta)} E_A \left[ c_2^{1-\gamma} \right]^{\frac{\beta}{\gamma}},
\]

and similarly for state B. Given this measure, how should we evaluate the agent’s welfare at period 0 when the agent is not yet born and the state in period 1, A or B, is still uncertain? One option is to simply take the expected value of time-1 utility, namely

\[
\hat{U}_0 = .5U_1^A + .5U_1^B.
\]

We call this measure expected ex-interim utility. Now suppose \( c_1^A = c_2^A = 1 \) (implying \( U_1^A = 1 \)) and \( c_1^B = c_2^B = 3 \) (implying \( U_1^B = 3 \)) and consider a policy that, if introduced at time zero, will deliver \( c_1 = c_2 = 2 \) (implying \( U_1 = 2 \)) for sure. The welfare measure (13) implies indifference with respect to that policy. But why should uncertainty with respect to an agent’s state at birth be evaluated risk-neutrally? In principle, any arbitrary degree of risk aversion with respect of the state of being born can be assumed. Blanchard’s approach, which entails evaluating this risk based on a risk aversion coefficient of 1, simply amounts to\footnote{Or in homogeneous form
\[
U_0 = \exp(\log(U_1^A) + \log(U_1^B)).
\]}

\[
U_0 = \log(U_1^A) + \log(U_1^B).
\]

Our alternative is to evaluate the uncertainty about the state in which an agent is born with the same degree of risk aversion with which the agent evaluates the uncertainty of old-age consumption. This leads to our ex-ante welfare measure

\[
U_0 = \left( (U_1^A)^{1-\gamma} + (U_1^B)^{1-\gamma} \right)^\frac{1}{1-\gamma},
\]

which accords with EZ preferences as we now show.
Derivation of Ex-Ante Utility

We define Epstein-Zin utility recursively in a way that makes it homogeneous of degree one:

\[ U_\tau^t = v^{-1} \left( v(c_\tau) + \beta v \left( u^{-1} \left( E_\tau \left[ u \left( U_{\tau+1}^\tau \right) \right] \right) \right) \right) \text{ for } t \leq \tau < T, \text{ and } U_T^t = c_T, \]

with \( u \) capturing the attitude towards risk and \( v \) representing the attitude towards intertemporal substitution. We take \( u(x) = x^{1-\gamma}/(1-\gamma) \) and \( v(x) = \log(x) \). To determine ex-ante welfare of a generation born at \( \tau < t \) we assume that agents born at \( \tau \) neither derive utility from consumption nor discount the future before birth – the same assumptions we would make in the time-separable case. Consequently, the utility of a generation born at \( t \), evaluated at, say, \( \tau = t - 2 \) becomes

\[ U_{t-2}^t = (v^{-1} \circ v \circ u^{-1}) \left( E_{t-2} \left[ \left( u \circ v^{-1} \circ v \circ u^{-1} \right) \left( E_{t-1} \left[ u(U_{t+1}^\tau) \right] \right) \right] \right) = u^{-1} \left( E_{t-2} \left[ E_{t-1} \left[ u(U_{t}^\tau) \right] \right] \right). \]

By iteration, the utility of a generation born at \( t \) evaluated \( \tau = 0 \) is

\[ U_0^t = u^{-1} \left( E_0 \left[ u(U_{t}^\tau) \right] \right), \]

which amounts to (10) when using our specific utility function.

4 Closed-Economy Findings

This section presents four sets of closed-economy results arising from defined-benefit pay-go and related policies. Each exercise demonstrates the crucial importance of risk-sharing to long-run ex-ante utility and, thus, to the prospects for an EAU Pareto improvement. First, we explore the potential for long-run EAU increases under what we view as highly favorable calibrations. We focus in detail on two baseline calibrations. For each, we decompose the EAU impact of DB pay-go showing that risk-sharing is the source of long-run EAU gains when they arise. It is also a mitigating factor when long-run EAU falls. Second, we show that running what, on average, is the same size deficit, but doing so in ways that entail different risk-sharing can materially change the EAU impact of deficit finance. Third, we show that a policy of two-way transfers, entailing no intergenerational redistribution, produces larger EAU gains than does DB pay-go. Fourth, we show that Blanchard’s assumed endowment, with its risk-sharing capacity, is key to a Pareto improvement when it arises.

\[ U_0^t = E_0 \sum_{\tau=t}^{t+T} \beta^{\tau-t} u(c_\tau), \]

where a per-period utility function \( u \) captures both the attitude towards risk and intertemporal substitution.

\[ \text{Time-separable utility evaluated at time zero would be} \]

\[ U_0^t = E_0 \sum_{\tau=t}^{t+T} \beta^{\tau-t} u(c_\tau), \]
Table 1: The impact of defined-benefit pay-go policy on long-run ex-ante utility for different calibration targets for the risk-free rate and the risk-premium.

<table>
<thead>
<tr>
<th>RP \ RFR</th>
<th>0.0%</th>
<th>-1.0%</th>
<th>-2.0%</th>
<th>-3.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.0 %</td>
<td>-1.8%</td>
<td>-0.7%</td>
<td>+1.6%</td>
<td>+6.0%</td>
</tr>
<tr>
<td>3.0 %</td>
<td>-2.0%</td>
<td>-1.3%</td>
<td>+0.2%</td>
<td>+3.2%</td>
</tr>
<tr>
<td>4.0 %</td>
<td>-2.1%</td>
<td>-1.7%</td>
<td>-0.7%</td>
<td>+1.3%</td>
</tr>
</tbody>
</table>

4.1 Role of the Risk-Free Rate, Risk-Premium, and Policy Scale

We now consider the EAU impact of introducing DB pay-go policy. Following Blanchard (2019), each young cohort pays the old a fixed amount, set at 20 percent of average capital in the no-policy, stochastic steady state. Unless otherwise stated, all results presented below reflect policies of this size. Since both the current young and current old clearly gain from an introduction of these transfers, increases in EAU for generations born in the long run, which we call long-run EAU, indicates, as one would expect and we confirmed, a Pareto improvement – thus, if long-run EAU increases, EAU for all generations rises. Obviously, if long-run EAU falls, the policy is not Pareto efficient. Hence, an increase in long-run EAU indicates a Pareto improvement, whereas a decline signals assisting some generations, while hurting others.

Table 1 reports, for different risk-free-rate (RFR) and risk-premium (RP) calibrations, the percentage impact on long-run EAU. Of the five cases with gains, only one, featuring a RP of 3 percent and a RFR of -2 percent, is, in our view, remotely plausible. We next examine two cases, denoted Baseline 1 (B1) and Baseline 2 (B2), in more detail. Both feature a relatively low RP of 3 percent. B1 calibrates preferences to a -1 percent RFR, while B2 calibrates preferences to a -2 percent RFR. For B1, expected utility of those born in the long-run falls by 1.3 percent. For B2, it rises by 0.2 percent.

Policy scale plays an important role in determining long-run EAU impacts. As the left-hand-side (LHS) of figure 1 shows, the percentage change in long-run EAU is negative under B1. But for B2, it starts positive and goes negative at a policy scale equal to roughly 25 percent of the long-run, no-policy, average capital stock. That’s not much larger than the 20 percent value considered by Blanchard (2019). But this policy transferring 20 percent of capital corresponds to only a 3 (or 4) percent tax on the young’s income in B1 (or B2).¹³ That’s far below the combined explicit and implicit average wage-tax rate used to finance U.S. intergenerational redistribution. Figure 1 shows major expected utility losses under both B1 and B2 as the economy moves from running DB pay-go based on a fixed transfer that ranges in magnitude from zero to 25 percent of the young’s income – the no-policy, long-run average of their wages plus endowment, to be precise.

Figure 2 shows EAU effects of the transfer scheme on both current and prospective generations. Clearly, the initial old gain, as they simply receive a transfer with no strings attached. Their EAU, which we omit from figure 2, increases by 1.8 percent and 2.9 percent in B1 and B2.

¹³Recall that the period length is 25 years. So the yearly transfer is less than 1 percent of aggregate capital.
Figure 1: Long-run EAU impact of DB pay-go, measured as a fraction of aggregate capital (LHS) and young’s total income (RHS).

Figure 2: Generation-specific EAU impact along the transition path; initial conditions equal long-run averages.

respectively. The current young also gain substantially, because crowding out takes effect only after they are old. Hence, their wages when young are unchanged, but the returns they earn when old on their savings are higher due to the smaller amount of capital they, as a generation, bring into old age.
4.2 Role of Risk-Sharing

To clarify how deficit policy works, we now decompose changes in EAU into non risk-sharing and risk-sharing effects.\(^1\) We begin with the risk-neutral effect (RNE) referenced above. It captures the change in EAU that would arise for risk-neutral agents with an IES of 1. Their utility function is defined as follows.

\[
U^t_0 = E_0 \left[ c^{1-\beta} \cdot E_t \{ c_{o,t+1} \}^{\beta} \right].
\]  

(16)

The RNE, the ratio between the agent’s utility before and after the introduction of a transfer, picks up the DB pay-go policy’s crowding out of capital, which in all of our calibrations leads, on average, to lower long-run levels of consumption both when young and old.\(^2\) In addition, DB pay-go policy tilts the life-cycle age-consumption profile towards old-age consumption, which is also captured by the risk-neutral effect as it incorporates the agent’s IES.

We argue that risk-sharing comes in two distinct forms, one relating to the riskiness of old-age consumption given the date-event of birth, the other to the state into which generations are born. Recall, the former is called the life-cycle-risk effect (LRE) and the latter the cohort-risk effect (CRE). We define LRE as a change in \( \hat{U}^t_0 / \bar{U}^t_0 \), where the expected ex-interim utility of a generation is given by

\[
\hat{U}^t_0 = E_0 \left[ c^{(1-\beta)}_y \cdot E_t \{ c_{o,t+1}^{1-\gamma} \}^{\frac{\beta}{1-\gamma}} \right].
\]  

(17)

As our simple example above clarifies, (17) doesn’t capture uncertainty over the state into which one is born. This brings us to CRE, which is defined as the change in \( U^t_0 / \hat{U}^t_0 \). To sum up, we write ex-ante utility (EAU) as the product of three terms,

\[
U^t_0 = \bar{U}^t_0 \cdot \hat{U}^t_0 \cdot \frac{\hat{U}^t_0}{U^t_0},
\]

(18)

so that percentage changes in EAU equal, to a first order, the sum of percentage changes in the three terms – RNE, LRE, and CRE.

\[
\frac{\Delta U^t_0}{U^t_0} \approx RNE + LRE + CRE.
\]

\(^1\)For his part, Blanchard (2019) decomposes welfare changes from DB pay-go as arising from 1) providing agents with a higher safe return than is paid by the safe asset and 2) the crowding out of capital. Blanchard’s equation 3 captures this first effect. His discussion suggests this effect is positive if the safe rate is less than 1. That’s true for the first generation making the transfer. But one needs to average this term over future states of the economy to understand its contribution to the EAU of future generations. Doing so indicates that the expected value of this term equals the sum of a) the product of the average value of \( X \) (the difference between 1 and the risk-free rate) and \( Y \) (the average value of the marginal utility of second-period consumption) and b) the covariance of \( X \) and \( Y \). Both terms depend on risk-sharing arrangements. Hence, Blanchard’s decomposition confounds the impact of DB pay-go policy on risk-sharing with changes in average consumption values.

\(^2\)Barbie et al. (2007) provide conditions on prices that ensure that a reduction in investment increases aggregate consumption – conditions that don’t hold in our model.
Figure 3: Decomposing the impact of DB pay-go policy on long-run ex-ante utility.

Figure 3 decomposes our DB pay-go policy’s long-run EAU changes into their RNE, LRE, and CRE components. The RNE effect is, thanks to the model’s crowding out, negative – increasingly so with policy scale. The two risk-sharing effects, RNE and LRE, are, on the other hand, both positive, the LRE effect being stronger. RNE, LRE, and CRE sum to the overall impact – the solid blue curve. The reason that curve is not lower, in B1, and positive, for a range, in B2 is thus clearly due to risk-sharing. In both cases the risk-sharing effects and, as a consequence, the overall effect is concave, which is why in B2 the welfare impact exhibits an interior maximum, around 10 percent, and eventually turns negative, around 25 percent.

4.3 Role of Deficit-Policy Type

All deficit policies aren’t created equal. Blanchard’s DB pay-go policy improves risk-sharing by maintaining transfers to the elderly regardless of the economy’s state. This transfers risk to those who can best bear it – the young, thanks to their fixed endowment. We consider two alternatives. First, a defined-contribution (DC) pay-go policy: \( T_t = \kappa w_t \), for a fixed \( \kappa \). Second, a policy that Blanchard (2019) considers as maintaining a constant level of debt, \( D \). This policy entails \( T_t = DR_t^f \) and will be called constant debt (CD) in what follows. Both alternatives are calibrated such that the long-run average transfer from the young to the old equals that under DB.

As table 2 shows, DC pay-go generates a long-run EAU loss under both B1 and B2, which can be decomposed as follows: DC produces smaller RNE losses and larger CRE gains compared to DB. But the LRE gains are zero under DC, whereas they are large under DB as the sure transfer in DB reduces the risk of old-age consumption. On balance, these factors make DC...
policy substantially worse than DB policy. As for constant debt policy, table 2 shows that it slightly dominates the DB policy in terms of its long-run EAU impact. This is, as table 2 reveals, entirely due to the improved CRE. CD improves risk-sharing between cohorts as it offers generations born with a bad shock a larger transfer when old than generations born with a good shock. This is because the constant debt scheme’s transfer to the old in \( t + 1 \) is proportional to the interest rate \( R_{t+1} \) (accruing from \( t \) to \( t + 1 \)), which is higher when there is a bad TFP shock in \( t \). To summarize, the risk-sharing properties of deficit finance clearly matters to its EAU impact.

4.4 Risk-Sharing Through Two-Way Transfers

Given the crucial role of risk allocation, we now construct a risk-sharing scheme that does a far better job allocating risk. Our revised policy transfers from the young to the old if there are two below-median TFP shocks in a row. The old alive after two such shocks are hit with a lifetime double whammy – a particularly low wage when young and a particularly low return to capital when old. In alternative cases, when there are two above-median TFP shocks in a row, the revised policy transfers from the old to the young. In all other states, there is no transfer. Note that this implies that the average net transfer across generations is zero. We refer to this as the Two-Way Transfer (TT) scheme.

Under TT, the risk-free rate averages, on an annualized basis, -0.40 percent (-1.28 percent) in B1 (B2). In comparison, the mean of the RFR is 0.57 percent (-0.15 percent) in periods when transfers from the young to the old are positive and -1.32 percent (-2.33 percent) when they are negative. This substantial difference reflects three factors. First, low saving and investment in the prior period when the TFP shock was bad. Second, low saving and investment in the current period when the TFP shock is bad. These effects are both present even without policy, yet there is now a third mechanism that reinforces the resulting counter-cyclicality of interest rates – the reduced demand for the safe asset by the young who know they will receive a transfer when old if another bad TFP shock is to come. Thus, loose fiscal policy should coincide with high RFRs and tight fiscal policy should coincide with low RFRs. Stated differently, a high, not a low RFR is the time to run deficits and a low, not a high RFR is the time to run surpluses.

The fact that low TFP coincides with high expected TFP growth and thus, all else equal, high interest rates is, in turn, a direct consequences of assuming i.i.d. realizations of the level of TFP. Therefore, the finding that CD dominates DB has to be taken with a grain of salt.

---

**Table 2: Long-run EAU impacts from defined-benefit (DB), defined-contribution (DC), and constant-debt (CD) policies. Decomposition into risk-neutral effect (RNE), lifecycle-risk effect (LRE), and cohort-risk effect (CRE).**

<table>
<thead>
<tr>
<th>Case</th>
<th>RFR</th>
<th>RP</th>
<th>Overall Effect</th>
<th>RNE</th>
<th>LRE</th>
<th>CRE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>DB</td>
<td>DC</td>
<td>CD</td>
<td>DB</td>
<td>DC</td>
<td>CD</td>
</tr>
<tr>
<td>B1</td>
<td>-1.0%</td>
<td>3%</td>
<td>-1.3 -1.6 -0.9</td>
<td>-3.2</td>
<td>-2.3</td>
<td>-3.3</td>
</tr>
<tr>
<td>B2</td>
<td>-2.0%</td>
<td>3%</td>
<td>0.3 -1.0 0.9</td>
<td>-2.7</td>
<td>-1.9</td>
<td>-2.7</td>
</tr>
</tbody>
</table>

16The fact that low TFP coincides with high expected TFP growth and thus, all else equal, high interest rates is, in turn, a direct consequences of assuming i.i.d. realizations of the level of TFP. Therefore, the finding that CD dominates DB has to be taken with a grain of salt.
Tables 3 compares and decomposes long-run EAU impacts of DB pay-go, TT, and a combination of the two (TT+DB): the TT scheme plus a DB pay-go plan whose transfer is fixed at 5 percent of the no-policy, long-run average capital stock. TT produces a Pareto improvement for both B1 and B2. Indeed, the long-run EAU impacts of TT policy are substantially larger than under DB pay-go. Moreover, adding even a small DB pay-go policy on top of the TT policy reduces the values of both B1 and B2 long-run EAUs. In other words, implementing DB pay-go in the context of TT policy makes early generations better off and future generations worse off.

Figure 4 compares DB, TT, and TT+DB and shows their impacts on EAU of current and future generations along the transition. Clearly, a transfer scheme designed to share risk is far more efficient than Blanchard’s DB deficit finance, which requires just the right parameters and just the right scale to share risk.

Figure 5 varies the size of the risk-sharing scheme and decomposes the resulting long-run EAU impacts into our three effects. Compared to the DB pay-go scheme, the negative risk-neutral effect is much smaller in size. This is as expected given that two-way transfers entail no systematic redistribution from the young to the old and, therefore, no systematic crowding out. As for the life-cycle-risk effect, it slightly improves relative to the DB pay-go case. The cohort-risk effect is smaller or similar depending on the calibration. Thus, this policy achieves a similar level of risk-sharing between cohorts with much less crowding out, resulting in much larger increases in EAU overall.

Our two-way transfers scheme is certainly sub optimal. The optimal scheme would surely depend non-linearly on the economy’s state vector – its TFP and stock of capital. But the above analysis demonstrates the ample room for Pareto-improving policy that doesn’t systematically redistribute. Moreover, as figure 4 shows, when even our crude bilateral risk-sharing scheme is in place, adding DB pay-go policy scaled at 5 percent reduces the EAU of future generations.

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17 We assume, as with the DB scheme above, that the scheme is introduced in a period with a median realization of the TFP shock which implies that there is no immediate transfer. In the subsequent period a below-median or above-median shock triggers half the usual transfer. The current old are thus unaffected. But the young gain because they are, in effect, given an asset for free that hedges against their own old-age consumption risk.
Figure 4: Generation-specific EAU impact of different policies along the transition path; initial conditions equal long-run averages.

Figure 5: Decomposing the impact of two-way transfer (TT) policy on long-run EAU.

Indeed, only the initial old benefit from the addition of Blanchard’s policy. This result, again, indicates that deficits are desirable only insofar as they help share risk across generations. If risk is already well shared, intergenerational redistribution, whether run under the heading deficit finance, structural tax change, pay-go social security, or something else, will benefit
early generations at a cost to future generations.

### 4.5 Role of the Endowment

While wages and capital income are perfectly correlated via the TFP shock, the fixed endowment, in the context of 100 percent depreciation of capital in each period, makes the resources of the young (the endowment plus their wages) less risky than that of the old (their capital income). Unfortunately, the young can’t, in their infancy, make risk-sharing deals with their parent’s generation. This creates a missing market, which the government can implicitly supply via policy.

To quantify the endowment’s importance, we simulate DB pay-go in the closed economy but without the endowment.\(^\text{18}\) Table 4 shows the long-run EAU impact from implementing DB in the no-endowment economy for different calibration targets. Cell-specific results for the economy with the endowment are in brackets. The long-run EAU impact is positive in only two cases – with a zero or negative risky return. Otherwise, there are welfare losses that are often significant. Compared to the model with the endowment, the long-run EAU impact is roughly two percentage points worse for most calibrations.

To illustrate the very different message that the economy without the fixed endowment sends, figure 6 plots the long-run EAU impact as a function of the calibration target for the risk-free rate. It does so in three ways. We first reconsider the economy with the fixed endowment and lower risk-free rates (going from right to left), while keeping the risk premium calibration target fixed. This curve, despite being mostly in negative territory, sends the message that transfers are more desirable the lower the risk-free rate. This message changes however, when, instead of the risk premium, the risky rate is held fixed (which is achieved by increasing the risk-premium via an increase in the risk-aversion parameter). In this case the curve becomes substantially flatter and does not reach positive territory even for a risk-free rate of minus three percent. Finally, we move to the economy without a fixed endowment. Now reducing the risk-free rate no longer improves long-run EAU whatsoever. In short, a low risk-free rate does not, per se, represent a general invitation to run deficits.

\(^{18}\)In so doing, we truncate the TFP shock to avoid potential transfer-scheme collapse. Truncating at nine standard deviations suffices for this purpose. Second, we adjust capital’s share in the production function to ensure that the new model’s capital share matches Blanchard’s effective capital share. This adjustment doesn’t materially affect our results.
5 Open-Economy Findings

As is clear, the reason deficit finance makes future generations worse off, under reasonable calibrations, is its crowding out of capital. But in an open-economy, domestic saving reductions are spread globally. This limits domestic crowding out, leaving deficit finance more leeway to Pareto-improve ex-ante utility of domestic residents – current and future – via enhanced risk-sharing. However, domestic Pareto gains come at a price to foreigners as they have less capital with which to work and, thus, earn lower wages. In this case, putative domestic Pareto improvements partly reflect beggar-thy-neighbor policy. To assess this issue we add a foreign country to our model. TFP-shocks in the two countries are still assumed to have no autocorrelation. The cross-correlation is assumed to be positive, yet substantially below one. We first consider the case of two symmetric countries. We then model the U.S. as the home country and the rest of the world as the foreign country.

5.1 Two Symmetric Countries

As in the closed economy, we assume that country-specific TFP-shocks have a standard deviation of 20 percent. The cross-correlation of the two shocks is assumed to be 0.25.\textsuperscript{19} Throughout

\textsuperscript{19}This is close to the value of 0.22 calibrated in section 5.2.
Table 5: Comparing long-run EAU impacts and their decomposition from defined-benefit pay-go policy in closed and open economies.

<table>
<thead>
<tr>
<th>Case</th>
<th>Closed</th>
<th>Open</th>
<th>Closed</th>
<th>Open</th>
<th>Closed</th>
<th>Open</th>
<th>Closed</th>
<th>Open</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>H</td>
<td>H</td>
<td>F</td>
<td>H</td>
<td>H</td>
<td>F</td>
<td>H</td>
<td>H</td>
</tr>
<tr>
<td>B1</td>
<td>-1.3</td>
<td>-0.3</td>
<td>-0.3</td>
<td>-3.2</td>
<td>-1.5</td>
<td>-1.7</td>
<td>1.6</td>
<td>0.9</td>
</tr>
<tr>
<td>B2</td>
<td>0.3</td>
<td>1.4</td>
<td>0.2</td>
<td>-2.7</td>
<td>-0.6</td>
<td>-1.9</td>
<td>2.6</td>
<td>1.5</td>
</tr>
</tbody>
</table>

Table 5 shows what we expect: a smaller decrease (or larger increase in case of B2) in home-country, long-run EAU compared to the closed-economy case. This reflects the beggar-thy-neighbor effect. As the decomposition in table 5 indicates, the closed-open economy differences primarily reflect differences in the size of RNE. RNE is a smaller negative number in the open economy case under both baselines. But the improvement for the home country in RNE is particularly pronounced under B2. As for risk-sharing, both countries experience the same LRE effects and the foreign country benefits even more from the CRE effect than the home country. These reductions of life-cycle and cohort risk in the foreign country are driven by the bond market. In the no-policy case there is a zero cross-country bond position (due to symmetry), yet an introduction of the DB policy in the home country increases the world interest rate by 0.7pp (or 0.9pp in B2) and induces the

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Figure 7: Domestic and foreign long-run EAU impacts from domestic DB pay-go policy in the symmetric-countries case.

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this section, we retain our two baseline calibration targets.\(^\text{20}\) Table 5 shows what we expect: a smaller decrease (or larger increase in case of B2) in home-country, long-run EAU compared to the closed-economy case. This reflects the beggar-thy-neighbor effect. As the decomposition in table 5 indicates, the closed-open economy differences primarily reflect differences in the size of RNE. RNE is a smaller negative number in the open economy case under both baselines. But the improvement for the home country in RNE is particularly pronounced under B2. As for risk-sharing, both countries experience the same LRE effects and the foreign country benefits even more from the CRE effect than the home country. These reductions of life-cycle and cohort risk in the foreign country are driven by the bond market. In the no-policy case there is a zero cross-country bond position (due to symmetry), yet an introduction of the DB policy in the home country increases the world interest rate by 0.7pp (or 0.9pp in B2) and induces the

\(^{20}\)Since an equally weighted portfolio of home and foreign stock is less risky than before, we need to re-calibrate preferences to hit the targets. For B1, $\beta = 0.93$ and $\gamma = 30.28$, for B2, $\beta = 0.94$, $\gamma = 30.58$. 

21
foreign country to build up a sizable bond position – roughly 30 percent of its GDP. In this way the bond market allows the two countries to share the risk-mitigation benefits of the home country’s DB policy. This is similar to the capital market’s spreading the burden of crowding out across the two countries.

To take a closer look at the mechanisms involved, we now vary policy scale. Figure 7 considers the long-run EAU impacts of domestic DB pay-go schemes of different sizes. As the left-hand panel shows, for B1, agents in the home country are now better off if the policy is small in scale. But the gain to domestic agents comes at a loss to foreign agents. Clearly, the home country benefits from the positive effect of the transfers while the negative crowding out effects are shared, roughly equally, by both countries. Since the two countries are of equal size, domestic crowding out remains significant. For DB pay-go policy larger than 12 percent, this
crowding out dominates positive risk-sharing effects, reducing domestic long-run EAU. Under B2, in contrast, enhanced international risk-sharing arising from domestic DB pay-go leaves both countries better off.

The importance of international risk-sharing becomes clearer when we decompose long-run EAU effects. The upper two panels in figure 10 decompose the long-run EAU effects in the home and foreign countries for B1. While the long-run EAU effect for the foreign country is negative, due to crowding out, both risk-sharing effects are actually significantly positive. This is confirmed in the lower panel of the figure that considers B2. Here, both countries gain, with the welfare improvement clearly driven by risk-sharing. Since the home government is not redistributing among foreign generations the only source of the improved risk-sharing is international risk-sharing through the bond market.

To summarize, there are two effects at play in a two-country setting. First, the capital crowding-out induced by government transfers in the home country is shared with the foreign country. This limits the capital reduction in the home country while decreasing the capital stock in the foreign, permitting domestic EAU gains that come at welfare costs to the foreign country. But there is also enhanced international risk-sharing. The reduction in domestic saving reduces the domestic demand for safe bonds, lowering their price and raising their return. This permits foreign agents to achieve a safer portfolio at a lower cost. Still, achieving a global Pareto improvement requires invoking, in the case of B2, a rather low risk-free rate and an unrealistically low value for the risk-premium.

So far we have only considered DB pay-go policies in the home country, assuming there are no policies in place in the foreign country. Yet what happens if both countries run DB pay-go policies? If the two countries cooperate to jointly maximize their long-run EAUs, they will realize that the best policy is no policy (in the case of B1) or a moderate DB policy of 15% (in the case of B2). In a non-cooperative solution, however, national self-interest would lead both countries to use deficit finance to free ride on each other, resulting in symmetric DB policies of 4% and 18% in B1 and B2, respectively. The result is lower global capital and long-run EAU. Indeed, if both countries seek to maximize their long-run EAU’s taking the other country’s DB pay-go policy as given, the resulting Nash equilibrium entails roughly a 5 bp decline in EAU (under B1 and B2) relative to the cooperative solution. While this number is small, it would certainly be much bigger in a setting with more countries and thus more room for free-riding.

Finally, suppose the current young could choose the size of the transfers, which they pay when young and receive when old. The levels that maximize their utility, starting from an average state, are much higher than the levels that maximize long-run EAU. Interestingly, it is the cooperative solution among the home and foreign young (40% and 51% in B1 and B2) now delivers even higher levels of DB policy than the Nash equilibrium (25% and 37% in B1 and B2). In other words, if the current young act in concert, not to speak of the current old, who would certainly be on board, they can raise their welfare at the expense of future generations who are left with the burden of an ongoing, large pay-go system and the associated lower levels of capital and wages.
Table 6: Long-run EAU impact of domestic DB pay-go policy for different calibration targets.

<table>
<thead>
<tr>
<th>RP \ RFR</th>
<th>0.0%</th>
<th>-1.0%</th>
<th>-2.0%</th>
<th>-3.0%</th>
</tr>
</thead>
<tbody>
<tr>
<td>U.S. / RoW</td>
<td>U.S. / RoW</td>
<td>U.S. / RoW</td>
<td>U.S. / RoW</td>
<td></td>
</tr>
<tr>
<td>2.0 %</td>
<td>-0.6% / -0.1%</td>
<td>+0.4% / -0.0%</td>
<td>+2.4% / +0.2%</td>
<td>+6.3% / +0.6%</td>
</tr>
<tr>
<td>3.0 %</td>
<td>-0.7% / -0.1%</td>
<td>+0.0% / -0.1%</td>
<td>+1.5% / +0.1%</td>
<td>+4.3% / +0.4%</td>
</tr>
<tr>
<td>4.0 %</td>
<td>-0.7% / -0.2%</td>
<td>-0.2% / -0.1%</td>
<td>+0.8% / -0.0%</td>
<td>+2.9% / +0.2%</td>
</tr>
</tbody>
</table>

5.2 The U.S. and the Rest of the World

We now consider a more realistic calibration that treats the U.S. as the domestic country and the rest of the world (RoW) as the foreign country. We re-calibrate the model to match the relative sizes of the two economies, the volatility and correlation of their productivity shocks, and the sizes of international portfolio positions. Appendix C provides calibration details. The basic targets are as follows. Based on data from the Penn World Tables, RoW GDP is 6 times U.S. GDP and 1.25 times more risky. The correlation of the TFP shocks of the two countries is 0.22 and their auto-correlation is assumed to be zero.

As above, we choose identical preferences parameters for the two regions – parameters that match the risk-free rate and the equity premium in the home country, i.e. the U.S. The 2019 data from the Bureau of Economic Analysis reports RoW-held U.S. capital worth 80 percent of U.S. GDP, U.S.-held RoW capital worth 85 percent of U.S. GDP, and net U.S. bond holdings worth 40 percent of U.S. GDP.\(^{21}\) To fit these data, we introduce country-specific compensated investment costs that reduce the returns on cross-country investments. The terms \(\delta_H, \delta_G,\) and \(\delta_{FB}\) reference the cost to domestic agents of investing in foreign capital, the cost to foreigners of investing in home capital, and the cost to foreigners of investing in the bond. Our calibration uses these three parameters to match the three cross-country asset positions, \((k_{H,F}, k_{F,H}, b_H).\)^{22}

In B1 the annualized cost parameters needed to match the targets are: a 0.9 percent cost for the U.S. to invest in RoW capital, a 2.9 percent cost of RoW to invest in U.S. capital, and a 1.1 percent cost of RoW to invest in the international bond. With these costs, the associated risk-aversion coefficient needed to match the risk premium is 21.4.\(^{23}\)

Given this calibration, we now consider the long-run EAU consequences of a DB pay-go policy introduced in the home country, which turns out to be only slightly more favorable than in the closed economy case. As table 6 shows, realistic values of the risk-free rate and the risk-premium produce long-run EAU losses. The introduction of a transfer scheme in the U.S.

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\(^{21}\)Capital in the model corresponds to equity and foreign direct investment from the data, and bond holdings in the model correspond to net debt securities – debt securities held minus debt liabilities.

\(^{22}\)Given our assumed identical preferences in both regions, including unitary intertemporal elasticities of substitution, we have two preference parameters and three cost parameters to match the risk-free rate, risk premium in the home country, and the asset positions of the two countries in the context of global bond-market clearing, i.e., we have five parameters to match five moments.

\(^{23}\)Under B2, the calibrated cost parameters are slightly higher and the risk-aversion coefficient is slightly lower. While the calibrated cost parameters appear rather large – in particular, RoW investments in the U.S. cost almost 3 percent annually, there are certainly institutional features that make it difficult for most foreigners to directly invest in U.S. capital.
has negative long-run EAU effects for the rest of the world for realistic values of the return to capital. At the same time this slightly alleviates the negative effect in the home country. There are now several cases (e.g. a risk free rate of -1 percent with a risk premium of either 2 or 3 percent) where the home country enjoys (modest) long-run EAU gains whereas the foreign country experiences losses. In this case, the beggar-thy-neighbor effect is strong enough to imply long-run EAU gains in the home country that would not materialize in a closed economy. Finally, enhanced international risk-sharing can actually lead to situations where both home and foreign country gain, albeit for highly unrealistic values of the return to capital.

To illustrate this in more detail, we now consider the two baseline calibrations from our closed-economy analysis. The left-hand panel of figure 9 confirms the intuition. While in a closed economy, long-run EAU gains were impossible under B1, the home country now experiences small gains, and yet the rest of the world experiences losses. The right-hand panel of figure 9 illustrates that in B2, domestic long-run EAU improves significantly for a large range of transfer payments, while the rest of the world now also gains.

This result is not too surprising given the observations from the case of two symmetric countries. The crowding out of capital in the home country is further mitigated by the fact that the RoW is modeled as so much larger. Figure 10 confirms this. The figure’s upper half decomposes the long-run EAU effects for the B1 case. The risk-neutral effect is still negative for the home country, but it is dominated by the two positive risk-sharing effects for a large range of payments. Through enhanced international risk-sharing, the foreign country’s risk-sharing effects are also both positive. However, they are dominated by the risk-neutral effect and the overall effect is, therefore, negative. Under B2, as indicated in the lower half of figure 25.
10, the beggar-thy-neighbor effect even implies a positive risk-neutral transfer effect for the home country. The negative risk-neutral effect for the foreign country is overcompensated by enhanced risk-sharing. Consequently, the foreign country also gains, albeit very little.

Figure 10: Decomposing long-run EAU impacts of domestic DB pay-go policy in the asymmetric two-country model for calibrations B1 and B2.

To sum up, depending on calibration targets we observe three cases. For realistic returns to capital, domestic and foreign long-run EAU both decline. For very low returns to capital, both countries gain from deficit finance in the home country. For cases in the middle, including one of our baseline cases, the beggar-thy-neighbor effect implies home-country gains at the price of foreign-country losses.

As in the closed-economy calibration, we also consider the effects of alternative transfer schemes. Table 7 shows the long-run EAU effects for the two asymmetric countries of defined-
Table 7: Comparing long-run EAU impacts and their decomposition from different policies and calibrations in an asymmetric two-country calibration (U.S. as the home country and the RoW as the foreign country). Closed-economy results reported for comparison.

<table>
<thead>
<tr>
<th>Policy</th>
<th>Case</th>
<th>EAU change (in %)</th>
<th>RNE (in %)</th>
<th>LRE (in %)</th>
<th>CRE (in %)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Closed</td>
<td>Open</td>
<td>Cl. Open</td>
<td>Open</td>
</tr>
<tr>
<td>DB</td>
<td>B1</td>
<td>-1.3 0.04 -0.03</td>
<td>-3.2 -0.4 -0.5</td>
<td>1.6 0.2 0.3</td>
<td>0.4 0.2 0.2</td>
</tr>
<tr>
<td>DC</td>
<td>B1</td>
<td>-1.6 -0.90 0.08</td>
<td>-2.4 -1.7 -0.2</td>
<td>0.0 0.0 0.2</td>
<td>0.8 0.8 0.1</td>
</tr>
<tr>
<td>CD</td>
<td>B1</td>
<td>-0.9 0.01 0.01</td>
<td>-3.2 -0.4 -0.5</td>
<td>1.6 0.2 0.3</td>
<td>0.8 0.2 0.2</td>
</tr>
<tr>
<td>TT</td>
<td>B1</td>
<td>0.8 1.45 0.04</td>
<td>-0.9 0.4 -0.2</td>
<td>1.7 0.8 0.1</td>
<td>0.0 0.2 0.1</td>
</tr>
<tr>
<td>DB</td>
<td>B2</td>
<td>0.3 1.53 0.14</td>
<td>-2.7 0.8 -0.6</td>
<td>2.6 0.4 0.5</td>
<td>0.4 0.4 0.2</td>
</tr>
<tr>
<td>DC</td>
<td>B2</td>
<td>-1.0 -0.44 0.26</td>
<td>-2.0 -1.4 -0.1</td>
<td>0.0 -0.1 0.3</td>
<td>1.0 1.1 0.1</td>
</tr>
<tr>
<td>CD</td>
<td>B2</td>
<td>0.9 1.66 0.22</td>
<td>-2.7 0.8 -0.6</td>
<td>2.5 0.4 0.6</td>
<td>1.2 0.4 0.3</td>
</tr>
<tr>
<td>TT</td>
<td>B2</td>
<td>2.3 3.12 0.14</td>
<td>-0.8 0.9 -0.3</td>
<td>2.7 1.4 0.2</td>
<td>0.4 0.8 0.2</td>
</tr>
</tbody>
</table>

benefit, defined-contribution, constant debt, and two-way transfer policies. Compared to the closed-economy numbers (which are also provided in table 7), the results for all schemes are more favorable. This is due to the risk-neutral effect as the decomposition in table 7 reveals – and as was expected due to the dampening of crowding out due to openness.

Comparing different deficit-finance schemes, we find that long-run EAU gains for the home country are much harder to achieve for a defined contribution scheme. This mirrors our results for the closed economy. However, the rest of the world now experiences long-run EAU gains under DC. The key to understanding this is that under DC the transfers are perfectly correlated with home-capital returns, making home-capital less attractive for home investors; consequently, the crowding out effects are much stronger for the home country than for the foreign country as can be seen from the RNE values in table 7. With crowding out being modest in the foreign country, risk-sharing effects dominate and the overall impact on foreigners is positive. When it comes to public debt, we observe that its home country impact now compares less favorably to the defined benefit scheme than in the closed economy. For the foreign country, however, CD is substantially better than DB.

Finally, in order to underscore the point that long-run EAU gains in this model are about enhanced risk-sharing, we consider the two-way transfer scheme from above (Appendix D provides the decomposition as a function of policy size). For both calibrations, all three effects are positive for the home country. The life-cycle-risk effect is the most important. The foreign country also gains under both baselines. While the risk-neutral transfer effect is negative for the rest of the world, this is compensated by enhanced risk-sharing. As in several of the examples provided above, the beggar-thy-neighbor effect is present, but if the transfers are designed to enhance risk-sharing, they also provide better risk-sharing for the rest-of-the world, which, in this case, overcompensates.
6 Limitations of the Model

Following Blanchard (2019), we’ve examined a major policy question in the simplest of settings. There is no heterogeneity within cohorts, no borrowing constraints, no initial policy, no existing distortions, no externalities, no public goods problems, a limited ability, due to period length, for overlapping cohorts to share risk, no existing policy, and no monetary policy, let alone monetary policy in the context of a zero lower bound on the nominal interest rate.

Some of these factors connect directly to the issues discussed here. Of perhaps most importance is borrowing constraints, as stressed by Brumm et al. (2020), close to 90 percent of Americans are in debt and their safe rates – rates they can earn by pre-paying their mortgages, credit-card balances, student debt, etc. – are much higher than the U.S. average growth rate. This calls into question the basic premise that pay-go can pay for itself.

Moreover, America’s pay-go policy largely entails taxing workers’ earnings to subsidize the elderly. But as Altig et al. (2020) show, the scope for higher marginal taxation is limited. Consequently, pay-go, which, in practice, necessitates higher (lower) taxes on workers in bad (good) times will entail potentially far larger distortions since excess burdens rise with the square of tax wedges.

Heterogeneity and the interaction of micro and macro shocks can make economic life far riskier and raise the value of government-organized risk mitigation. Indeed, Harenberg and Ludwig (2019) suggests a greater role for DB pay-go risk-sharing in such a context. That said, if the problem lies with incomplete risk-sharing, as their paper argues, it should be addressed directly.

A model comprising a realistic number of periods, e.g., Hasanhodzic and Kotlikoff (2013), would provide scope for intergenerational risk sharing among generations that overlap and, therefore, less scope for deficit finance to Pareto improve.

Existing policy also matters. As we’ve shown, the impact on long-run EAU either steadily declines or rises and then declines in policy scale. Hence, calibrating and running Blanchard’s model assuming, as he does, that the U.S. has no initial policy in place appears to give DB pay-go an unwarranted efficiency advantage. Our demonstration that the DB pay-go policy, when introduced in the presence of existing risk-sharing, can no longer Pareto improve is telling in this regard.

Another caveat, which almost goes without saying, is that our focus, like Blanchard’s, is purely on government redistribution. We’re not considering deficit finance used to fund infrastructure or correct externalities, like global warming. Nor do we consider the value of deficits as counter-cyclical policy. Finally, Blanchard (2019)’s endowment assumption ensures that, once begun, intergenerational redistribution remains feasible through the end of time. But, to quote Dylan, tomorrow is a long time. Developed countries, with enough runway, can morph into underdevelopment. Argentina’s century-long transition from first- to third-world status is a case in point.
7 Conclusion

The decades-old deterministic literature on the potential over-accumulation of capital raises a green flag when the marginal product of capital, $m$, falls below the growth rate, $g$. The flag signals the ability to perpetually take from the young and give to the old, making all generations better off. Although measurements differ, there is, unfortunately, no doubt that $m > g$.

With uncertainty, the return to safe assets – the safe rate – allegedly comes into play. The U.S. safe rate routinely runs below the U.S. growth rate. This has led many economists to treat the average difference between the two as an arbitrage opportunity – one that can also be exploited via ongoing pay-go policy. But growth isn’t for sure. Again, Argentina’s century-long transition from developed to developing country is an abject lesson in very bad things happening to very good countries for a very long time.

Of course, economics teaches us how to weigh the likely good against the unlikely bad and decide whether a given policy would, on balance, benefit some or all generations without harming others. Blanchard (2019)’s provocative paper suggests that conditions may be right, in the U.S. and other low-interest-rate countries, to run fiscal deficits at no cost. No cost means helping current old generations at no cost to their progeny. We examine the “deficits are free” proposition using Blanchard’s self same model, but with an important correction to his formula for ex-ante utility, which leaves even more room to find Pareto improvements from deficit finance.

Blanchard’s framework does, indeed, admit cases for which deficit finance can Pareto improve from an ex-ante perspective. But the calibrations required for such an outcome stretch reality. Even then, deficit finance needs to be conducted in just the right way to deliver what turns out, at best, to be modest efficiency gains. Under close inspection, it’s clear why low safe rates aren’t a clear invitation to run deficits. Low safe rates signal a strong demand for safety. This need is directly addressed via systematic risk-sharing, not intergenerational redistribution.

Blanchard (2019)’s model is a two-period, OLG model with aggregate risk. In such models, Pareto-improving, government-organized, intergenerational risk-sharing policy is ripe for the picking given the inability of the living to trade with the unborn. Our decomposition of policy impacts on ex-ante utility into risk-neutral and risk-sharing factors identifies the Pareto gains from risk-sharing. It also shows that intergenerational redistribution per se – the focus of Blanchard’s study – actually undermines the potential to Pareto improve. In contrast, even crude, government-organized intergenerational risk-sharing – transfers running from the young to the old and vice versa depending on the economy’s current and prior states – can materially improve current and future generations’ ex-ante utility. As for deficits, when they Pareto improve, they do so serendipitously – not because they are designed to solve the problem at hand, namely addressing missing private financial markets, but because they too can share risk under very special conditions. But, as we show, once proper risk-sharing policy is in place, deficit finance provides no further scope for sharing risk.

We also explore deficit finance in an international context. In such settings, the crowding out of domestic saving translates less than one for one into reduced domestic investment. This
limits the reduction in domestic wages, albeit at the price of lower foreign wages. On the other hand, moving to an open economy raises the prospect of mutually beneficial international risk sharing. Yet, here again, bilateral risk-sharing, in this case, both between generations and between countries, not systematic redistribution from the young to the old or from foreign to domestic residents, may be best suited for achieving a Pareto improvement. In sum, our paper’s message is clear. When interest rates go low, government risk-sharing, not deficit finance, may provide the path to economic efficiency.
APPENDIX

A  Comparison of EAU Measures

Figure 11 compares long-run EAU effects under both our and Blanchard’s EAU measures for our two baseline calibrations in the closed economy. While the effects are qualitatively similar under both measures, it is clear that with our measure, EAU effects are more favorable.

![Baseline 1: RFR = -1%, RP = 3%](#)

![Baseline 2: RFR = -2%, RP = 3%](#)

Figure 11: Comparing our EAU measure with Blanchard’s measure. Impact of DB pay-go policy in the closed economy for both baseline.

B  Robustness and Sensitivity

We follow Blanchard (2019) and assume a high standard deviation of TFP and search for a risk-aversion parameter that matches the desired risk-premium. Table 8 provides sensitivity analysis with respect to this calibration method. The table’s first three rows consider higher and lower values of $\sigma$, which implies lower and higher values of $\gamma$. Neither the B1 or B2 EAU results are much affected. The table’s fourth row considers a calibration with a disaster shock, specifically a minus five standard deviation drop in TFP, which occurs each period with a 1 percent probability. All other TFP realizations are slightly increased to keep average TFP constant. The long-run EAU results are slightly worse for this calibration. Interestingly, as Barro (2020)’s work and intuition suggest, the model now calibrates with much lower values of $\gamma$ the model – 6.6 and 6.7 in B1 and B2, respectively.
### Table 8: Sensitivity of long-run EAU Changes from a 20% DB pay-go policy in the closed economy for changes in the distribution of TFP-Shocks

<table>
<thead>
<tr>
<th>TFP shocks</th>
<th>Baseline 1</th>
<th>Baseline 2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma = 0.15$</td>
<td>-1.31%</td>
<td>0.25%</td>
</tr>
<tr>
<td>$\sigma = 0.2$ (baseline)</td>
<td>-1.31%</td>
<td>0.24%</td>
</tr>
<tr>
<td>$\sigma = 0.3$</td>
<td>-1.33%</td>
<td>0.17%</td>
</tr>
<tr>
<td>$\sigma = 0.2 + \text{disaster}$</td>
<td>-1.40%</td>
<td>-0.01%</td>
</tr>
</tbody>
</table>

C Calibrating the Asymmetric 2-Country Case

According to Penn World Tables data, the U.S. share of world GDP totaled 16.4% in 2017 – hence our assumption that the RoW is roughly six times larger than the U.S. We use the standard deviation of the growth rate of real GDP of the U.S. and RoW to approximate TFP risk. The standard deviation of the log difference in U.S. GDP growth is 1.95 log-percent, whereas that of the RoW is 2.50 log-percent. Therefore, we calibrate TFP of the RoW to be 1.25 times as risky as that of the U.S.. The same data produce our assumed cross-country TFP correlation of 0.22.

Finally, we introduce cross-country investment costs to match the 2019 cross-country asset positions reported by the Bureau of Economic Analysis. We adjust the model as follows. The net cross-country investment return for home investors, $\hat{R}_{F,t+1}$, and the cross-country investment return for foreign investors, $\hat{R}_{H,t+1}$, are now given by

$$
\hat{R}_{F,t+1} = R_{F,t+1} - \delta_F,
\hat{R}_{H,t+1} = R_{H,t+1} - \delta_H,
$$

where $\delta_F$ and $\delta_H$ reference the costs to domestic and foreign residents of investing in capital abroad. We also introduce a bond investment cost, $\delta_{FB}$, which foreigners face when investing in domestic bonds. $\hat{R}_{t+1}^f = R_{t+1}^f - \delta_{FB}$. Domestic investors faces no cost when investing in domestic bonds and there is, by assumption, no separate foreign bond market. The budget constraints of the home and foreign old are

$$
c_{o,H,t+1} = k_{H,H,t}R_{H,t+1} + k_{H,F,t}\hat{R}_{F,t+1} + b_{H,t}R_{t+1}^f + T_{t+1} + TAC_{tH},
c_{o,F,t+1} = k_{F,H,t}\hat{R}_{H,t+1} + k_{F,F,t}R_{F,t+1} + b_{F,t}\hat{R}_{t+1}^f + TAC_{tF},
$$

where $c_{o,H,t+1}$ is the time-$t+1$ consumption of the old in the home country, $k_{H,H,t}$ and $k_{H,F,t}$ are domestic and foreign capital investments made by the young at time $t$, $b_{H,t}$ is the time-$t$ purchase of bonds by the young, $T_{t+1}$ is the government transfer received by the domestic old at time $t+1$, and $TAC_{tH}$ are the transaction costs, which we assume are lump-sum rebatted. In equilibrium we have $TAC_{tH} = \delta_F k_{H,F,t}$. Analogous formulations hold for foreign households.
Figure 12 compares and decomposes the long-run EAU impacts of home DB pay-go policy on the home and foreign country for varying policy size.

Figure 12: Comparing and decomposing long-run EAU impacts on home and foreign country of DB pay-go policy.
References


Reis, R. (2021). The constraint on public debt when $r < g$ but $g < m$. Working paper, London School of Economics.


