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**SCIENCE**

## **The Deficit Is Not a Well-Defined Measure of Fiscal Policy**

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# The Deficit Is Not a Well-Defined Measure of Fiscal Policy

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Notwithstanding its widespread use, the government's deficit is not a well-defined measure of fiscal policy from the perspective of neoclassical economics; the equations of neoclassical models do not define the deficit. Rather than being a fundamental economic concept, the deficit is an arbitrary cash flow accounting construct with no necessary relation to the true stance of fiscal policy. Although the deficit is supposed to indicate how the burden of paying for the government's consumption is spread across different generations, actual changes in the measured deficit in the United States have had little if any relation to changes in the burden imposed by the government on different generations. The deficit's lack of definition is illustrated with a simple model, and the potential for misreading fiscal policy is discussed with U.S. fiscal policy in the 1980s as an example. In this article, creation of present value generational accounts are called for that would properly measure the intergenerational stance of fiscal policy.

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**A**LTHOUGH THE GOVERNMENT DEFICIT IS WIDELY USED AS a measure of fiscal policy, the deficit is not a well-defined concept from the perspective of neoclassical macroeconomics. Neoclassical macroeconomics can be distinguished from its primary competitor, traditional Keynesian macroeconomics, by its grounding in microeconomics. Neoclassical macroeconomics derives its predictions by aggregating the decisions of households and firms. The life cycle model of Modigliani *et al.* (1, 2) is the most renowned neoclassical macromodel and is the one that raises the greatest concerns about loose fiscal policy. However, in the life cycle model, as in any neoclassical macromodel, the equations of the model do not define the government deficit. Indeed, from the perspective of these models the deficit is an arbitrary accounting construct whose value depends on how the government chooses to label its receipts and payments. Since rational households and firms see through accounting labels, the predictions of neoclassical models are independent of the choice of accounting. Not only does the choice of accounting labels have no implications for actual fiscal policy in neoclassical models, but the reverse is also true: in neoclassical macromodels the government can conduct any sustainable fiscal policy while simultaneously choosing its accounting so as to report any size surplus or deficit it desires.

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## Intergenerational Transfers and Loose Fiscal Policy

In neoclassical macromodels fiscal policies, aside from changes in government consumption, have real effects because they either (i) alter economic incentives, (ii) redistribute within generations, or (iii) redistribute across generations. It is this third policy, intergenerational redistribution and its implications for saving and investment, that underlies the concern about loose fiscal policy.

Intergenerational redistribution occurs whenever a government policy increases the present values of payments less receipts (net payments) it makes to some generations at the expense of the present value of net payments to other generations. Consider for different generations what happens to the present value of lifetime payments less receipts when the government runs a policy it describes as a "tax cut," financed in the short run by what it calls a "deficit" and in the long run by what it calls a "tax" increase to pay the interest on the accumulated "debt." Older generations are made better off in present value by this policy. In the short run they have to pay less to the government, and at the time the "tax" increase occurs they may be dead, or if they are still alive, they will only have to make the larger payments for a relatively short period of time. Middle-age generations may also be better off; if the "tax" is assessed on labor earnings, middle-age generations make smaller payments to the government during the "tax cut" period, but may escape through retirement most or all of the subsequent larger payments arising from the "tax" increase. Younger generations will, in contrast, be worse off because paying the higher amount of "taxes" for most of one's life will more than outweigh (in present value) the temporarily lower payments. Future generations will also be worse off because they will face higher net payments throughout their lives. Thus the "tax cut" policy redistributes from young and future generations to initial middle-age and older generations.

According to the life cycle model, intergenerational redistribution from younger and future generations to older generations crowds out (lowers) national saving because older generations will increase their consumption by more than younger generations lower their consumption. The reason is that older generations have fewer years left to live and consequently have fewer years over which to consume the additional resources. Younger generations, on the other hand, spread their reduction in lifetime resources over more years; hence their response to a decline in the present value of net lifetime receipts is somewhat to lower their consumption this year, knowing they will also lower their consumption for many years in the future. Since each generation's marginal propensity to consume is an increasing function of its age, redistribution toward older generations through policies such as "tax cuts" implies an increase in current consumption and a crowding out of saving. Economists in the United States are rightfully concerned about this crowding out process. The

postwar U.S. saving rate is less than two-thirds the rate observed from 1900 to 1930 and less than half those of Japan, West Germany, and many other developed economies.

## A Two-Period Life Cycle Model

A simple two-period, one-commodity life cycle model with zero population or productivity growth is convenient to show both the concern about loose fiscal policy and the fact that the government's reported deficit bears no necessary relation to the stance of fiscal policy. To keep things simple, the agents in this model face no uncertainty; adding uncertainty, including uncertainty about future government policy, would not alter any of the conclusions about the deficit's lack of definition. In this model a period can be thought of as standing for roughly 40 years. At the beginning of each period a new generation size is born, and members of each generation live for two periods, their youth and old age. When individuals are young they work full-time, and when they are old they are retired. Each individual born at time  $t$  chooses how much to consume when young at time  $t$ ,  $C_{yt}$ , and how much to consume when old at time  $t+1$ ,  $C_{ot+1}$ , subject to the budget constraint given in Eq. 1.

$$C_{yt} + C_{ot+1}/(1+r_{t+1}) = W_t \quad (1)$$

In Eq. 1,  $r_{t+1}$  is the interest rate at time  $t+1$ . The equation states that the present value of consumption expenditure (the price of consumption is normalized to 1) over the life cycle equals the present value of lifetime resources which, in this model, is simply earnings when young,  $W_t$ . If the utility from consumption when young and old is specified as the simple function in Eq. 2 with  $0 < \beta < 1$ , the maximization of utility subject to Eq. 1 gives the demands for consumption when young and old written in Eq. 3.

$$U_t = C_{yt}^\beta C_{ot+1}^{1-\beta} \quad (2)$$

$$C_{yt} = \beta W_t \quad (3)$$

$$C_{ot+1} = (1-\beta)W_t(1+r_{t+1})$$

At the beginning of any time period the young have no assets. Hence, the capital stock in the economy at time  $t+1$  corresponds to the asset holdings of the elderly at time  $t+1$ . The assets of the elderly at time  $t+1$  equal the savings they accumulated when they were young at time  $t$ . This savings per elderly equals  $W_t - C_{yt}$ , which is simply saving out of first period labor earnings. This fact and Eq. 3 permit one to write capital per young worker at time  $t+1$ ,  $K_{t+1}$ , as

$$K_{t+1} = (1-\beta)W_t \quad (4)$$

To close the model assume that the economy's single good is produced according to the production function in Eq. 5 that relates output per worker at time  $t$ ,  $Y_t$ , to capital per worker,  $K_t$

$$Y_t = K_t^\alpha \quad (5)$$

Given the production function, profit maximization by representative firms implies the following expressions relating factor demands to factor returns

$$W_t = (1-\alpha)K_t^\alpha \quad (6)$$

$$r_t = \alpha K_t^{\alpha-1}$$

Substitution of the first equation in Eq. 6 into Eq. 4 yields a nonlinear difference equation determining the time path of the economy's capital stock

$$K_{t+1} = (1-\beta)(1-\alpha)K_t^\alpha \quad (7)$$

If  $\alpha$  and  $\beta$  are less than one, this model has a locally stable, nonzero stationary state capital stock denoted by  $K$ , where

$$K = [(1-\beta)(1-\alpha)]^{1/(1-\alpha)} \quad (8)$$

## Adding Loose Fiscal Policy to the Model

Consider now a government policy commencing at the time  $t$  that takes an amount  $H$  from each young person and gives an amount  $H$  to each contemporary old person. For young individuals born at time  $t$  their lifetime budget constraint is now

$$C_{yt} + C_{ot+1}/(1+r_{t+1}) = W_t - H + H/(1+r_{t+1}) \quad (9)$$

Holding the time path of the wage rate,  $W_t$ , and the interest rate,  $r_t$ , constant, this fiscal policy leaves generation  $t$  as well as all subsequent generations worse off; each generation from  $t$  onward gives up  $H$  when young and must wait until old age to receive  $H$  back. Hence, each generation from  $t$  onward loses, in present value, interest on the amount  $H$ . The first generation of elderly alive at time  $t$ , in contrast, benefits from this policy since they receive  $H$ , but do not have to pay it back. Their second period budget constraint is now

$$C_{ot} = (1-\beta)W_{t-1}(1+r_t) + H \quad (10)$$

With Eq. 9, rather than Eq. 1, holding,  $C_{yt} = \beta[W_t - Hr_{t+1}/(1+r_{t+1})]$ , and the capital stock at time  $t+1$  is given by Eq. 11 since the saving of the young at time  $t$  now equals  $W_t - H - C_{yt}$ ,

$$K_{t+1} = (1-\beta)W_t - H(1+(1-\beta)r_{t+1})/(1+r_{t+1}) \quad (11)$$

The new capital stock transition equation is

$$K_{t+1} = (1-\beta)(1-\alpha)K_t^\alpha - H(1+(1-\beta)\alpha K_{t+1}^{\alpha-1})/(1+\alpha K_{t+1}^{\alpha-1}) \quad (12)$$

The new stationary state capital stock,  $K'$ , is found by setting  $K_t = K_{t-1} = K'$  in Eq. 12. Denoting by  $r$  the initial stationary state value of the interest rate, the derivative of the stationary state capital stock with respect to  $H$  evaluated at  $H$  equals zero is given by

$$\delta K'/\delta H = -(1+(1-\beta)r)/[(1-\alpha)(1+r)] < 0 \quad (13)$$

Equation 13 indicates that this intergenerational transfer policy crowds out the economy's long-run capital stock. Of course, the crowding out process takes some time, and Eq. 12 determines the transition path from  $K$  to  $K'$  associated with an increase in  $H$ .

The intuitive explanation for this crowding out of capital formation is that the redistribution to the initial elderly generation of  $H$  at time  $t$  leads to an increase in their consumption by the amount  $H$  (see Eq. 10), whereas the young at time  $t$  reduce their consumption by an amount  $\beta Hr_{t+1}/(1+r_{t+1})$ , which is less than  $H$ . Hence, aggregate consumption is larger at time  $t$ , and since output at time  $t$  is given, aggregate saving and investment at time  $t$  declines. This explains why the capital stock is smaller at time  $t+1$  as a consequence of the policy, but why does the economy end up in a stationary state with a permanently reduced capital stock? The answer is that although each successive generation will consume less because of this policy, their reduced consumption will, at any point in time, not yet have fully offset the initial increase in consumption of the time  $t$  elderly; that is, at any point in time there will always be generations yet to come whose consumption has yet to be reduced by the policy. In addition, the reduction in capital at time  $t+1$  means a lower level of wages at time  $t+1$  (see Eq. 6), which feeds back into lower savings by the young at time  $t+1$ , and an even lower capital stock at time  $t+2$ , with the process converging to the permanently lower capital stock of the new stationary state.

## Deficit Delusion and the Arbitrary Nature of Fiscal Labels

In presenting this simple example of loose fiscal policy, care was taken not to use any fiscal language to label the payment of  $H$  by each young generation to the government and the receipt of  $H$  from the government by each old generation. It now remains to show that this policy can be conducted with the government reporting a balanced budget, a debt, or a surplus. In each case the real effects of the policy are identical, and the reported size of the debt has no relation whatsoever to the stance of fiscal policy.

First, take the case that the government labels the receipt of  $H$  from the young each period as "taxes" and the payment to the old each period as "spending on transfer payments." In this case the government would report a balanced budget each period, since "taxes" equals "spending" each period, despite the fact that the government is running a loose fiscal policy. Furthermore, the budget would remain in balance the looser the fiscal policy—that is, the larger is the value of  $H$ .

Next let the government (i) label its payment of  $H$  to the elderly at time  $t$  as "spending on transfer payments," (ii) label its receipt of  $H$  from each young generation as "borrowing," and (iii) label its net payment of  $H$  to each elderly generation at time  $s$  for all  $s > t$  as "repayment of principal plus interest in the amount of  $H(1+r_s)$ " less a "tax in the amount of  $Hr_s$ ." While each generation of elderly starting at time  $t$  still receives  $H$ , and each generation of young starting at  $t$  still pays  $H$ , with this new labeling the government's deficit at time  $t$  is  $H$ , and its stock of debt remains at  $H$  forever. To see this note that at time  $t$  the government "spending" is  $H$ , and its reported "taxes" are zero. Hence, the time  $t$  deficit ("spending" less "taxes") is  $H$ . At time  $s$ , for  $s > t$ , the government's "spending on transfer payments" is zero, but its "spending on interest payments" is  $Hr_s$ . Since its "taxes" are also  $Hr_s$ , its deficit (change in the debt) after time  $t$  is zero, and its debt remains permanently equal to  $H$ .

As a third case, let the government (i) label its payment of  $H$  to the elderly at time  $t$  as "spending on transfer payments," (ii) label its net receipt of  $H$  from each young person at time  $t$  and thereafter as "receipt of taxes in the amount of  $2H$ " less a "loan in the amount of  $H$ ," and (iii) label its net payment of  $H$  to each elderly person at time  $s$  for  $s > t$  as "spending on transfer payments in the amount of  $2H + Hr_s$ " less "receipt of principal plus interest in the amount of  $H(1+r_s)$ ." At time  $t$  the government will now report a negative deficit ("taxes" less "spending") of  $-H$ . And at time  $s > t$  the government will report a balanced budget, since "taxes" of  $2H$  plus "interest received" of  $Hr_s$  will equal "spending on transfer payments of  $2H + Hr_s$ ." Hence, the government will report a positive stock of assets, a surplus, of  $H$  at time  $t$  and, since its budget will be balanced in each period after  $t$ , the government's surplus (negative debt) will remain at  $H$ .

These three labeling cases show that a fundamentally loose fiscal policy can be conducted with the government reporting zero debt, positive debt, or negative debt. Furthermore, there is nothing to preclude the government from changing its labeling through time with the consequence that the same real policy could first be reported as generating a deficit, then be reported as generating a surplus, and finally be reported as being conducted on a balanced budget basis. Finally, there is no requirement that the labeling produce either a zero debt, a debt of  $H$ , or a surplus of  $H$ . To see this, consider again the labeling leading to the reporting of a surplus. If the government labels its net receipt of  $H$  from the young as "taxes in the amount of  $5H$ " less "a loan of  $4H$ ," and labels the net payment of  $H$  to the elderly at  $s > t$  as "spending on transfer payments of  $5H + 4Hr_s$ " less "receipt of principal plus interest in the amount of  $4H(1+r_s)$ ," the reported surplus will be  $4H$  rather than simply  $H$ .

Hence, the government, "balance budget amendments notwithstanding, can report any size surplus or debt while engaging in exactly the same economic policy. And individuals, since they care only about their budget constraints, not the government's choice of labels, will behave exactly the same regardless of the announced, as opposed to actual, stance of fiscal policy. One immediate implication of this point is that by simply changing accounting labels, a government can adhere to a balanced budget amendment without altering its real course of fiscal policy one iota.

## Social Security and U.S. Fiscal Accounting—An Example

As a concrete example of how arbitrary fiscal labeling affects the reported deficit, consider how the U.S. government labels Social Security. When workers pay money to Social Security the payments are called "taxes"; when they get money back in old age, the payments are called "government spending on transfer payments." Suppose, instead, that the government had chosen at the inception of Social Security (i) to label its Social Security receipts from workers as "borrowing," giving the workers a piece of paper entitled "Social Security Bond" in the process, and (ii) to label its Social Security payments to workers when they retire as "repayment of principal plus interest" less a "tax" equal to the difference between the Social Security benefit and the full return of principal plus interest on the workers' Social Security contributions. In the process of making these payments to retired workers, Social Security would take back the piece of paper labeled "Social Security Bond." This alternative choice of language (which corresponds to case ii above), although of no real difference to workers or retirees, would have totally altered the official deficit. Had the government used this alternative language, it would have reported annual deficits in the 1960s and 1970s, when Social Security was expanded, as high as \$600 billion and a huge, roughly \$1 trillion surplus in 1983 as the result of the 1983 Social Security Amendments (3).

One response to this Social Security example is that, unlike the purchase of official government bonds, the purchase of Social Security bonds, if we use that language, is not voluntary. While this is true, the fact that payments to Social Security are mandated by law is likely to have either minor economic implications or none at all. For households that are not liquidity constrained (cash constrained), the fact that they are forced to purchase Social Security bonds will not affect their consumption choices. For liquidity-constrained households, the forced purchase of Social Security bonds will affect their consumption. But a number of recent studies indicate that the overwhelming majority of U.S. households are not liquidity-constrained, and their forced annual purchase of Social Security bonds (forced contributions to Social Security) is not influencing the timing of consumption over their life cycle (4). Stated differently, the great majority of U.S. households appear to make consumption decisions based on the present value of their resources.

A second response to this Social Security example is, "Yes, the government can play labeling and related games with its receipts and payments without altering their present values, but, with some exceptions, it does not. Furthermore, it basically uses the same labeling or accounting conventions through time." But, how does one know? If labeling is entirely arbitrary, how does one know what the government's labeling convention is? In addition, the fact that some politicians in the 1930s used one set of words rather than another does not preclude economists from considering a different historic choice of words and, as a consequence, reaching the conclusion that, for example, we ran huge surpluses rather than huge deficits in the 1980s. In short, for purposes of economic

research on the stance of fiscal policy, the official federal deficit is simply one of a infinite number of equally arbitrary and, unfortunately, irrelevant time series.

## How Should We Measure Fiscal Policy?

If the size of government's debt bears no necessary relation to the stance of fiscal policy, how should one properly measure the tightness or looseness of fiscal policy? The answer to this question depends on the particular neoclassical economic model that one is using. According to the life cycle model, fiscal policy should be described in terms of its effects on the budget constraints of different generations. The generational budgets in Eqs. 9 and 10 fully describe the policy discussed above. Note that these budgets are invariant to the choice of fiscal labels; that is, regardless of the choice of fiscal labels, Eq. 10 states that the policy increases by  $H$  the present value of generation  $t-1$ 's (the old at time  $t$ ) consumption and decreases by  $Hr_s/(1+r_s)$  for  $s>t$  the present value of generation  $s'$  consumption (5).

The total of these changes in the present value of what each generation can consume, discounted to the present and summed over all current and future generations, equals the present value of the change in the government's consumption (6). In our example, the change in the present value of government consumption is zero. Hence, the increase in the present value of one generation's consumption is offset by decreases in the present value of some other generations' consumption. Thus, by examining generational budgets one can see how different generations share the burden of paying for the government's consumption. Tight intergenerational fiscal policy corresponds to policy that places a larger share of the burden of paying for the government's consumption (which may be zero as in the current model) on current as opposed to future generations. Stated differently, tight intergenerational policy is policy that redistributes toward earlier generations.

## Subtle Intergenerational Policy

It is important to realize that the present value of what a generation can afford to consume, its budget constraint, can be affected in very subtle ways by the government. Perhaps the most subtle way in which the government can change the budget constraints of different generations is through policies that alter the market value of assets. Consider, for example, what happens when the government announces new investment incentives. Investment incentives are subsidies given to the production of new capital. Old capital, capital that was produced before the enactment of the subsidies, is not eligible for such subsidies. Hence, old capital is at a tax disadvantage in comparison with newly produced capital, and, as a consequence, the market price of old capital must fall. In concrete terms, the price of a new computer that is eligible for an investment subsidy will be higher than the price of an old computer that is ineligible even if the computers are physically identical. By enacting investment incentives, the government produces a one-time drop in the value of existing (old) assets. Since the elderly are the primary owners of existing assets, a decline in the market value of assets reduces the present value of what the elderly can consume. In contrast, the decline in the market value of assets expands the budget constraints of young and future generations because they can now purchase the existing assets at a lower price. If one added investment incentives to the simple model presented above, one would see that the introduction of investment incentives is structurally very similar to the reverse of the policy examined above (7).

## Reagan Fiscal Policy from the Perspective of Generational Accounts

From the perspective of generational accounts the Reagan Administration's fiscal policy, at least through 1986, appears, on balance, to have been fairly tight. Younger generations lost, on average, about \$12,000 in present value from the 1983 change in Social Security legislation. This loss was roughly equal to their present value gain from the income tax cuts. Older generations were not greatly affected by the 1983 change in Social Security and did benefit from the income tax cut; but the 1981 increase in investment incentives associated with the accelerated cost recovery system redistributed away from the elderly, and, at least through 1986, largely offset their gains from the income tax cut.

In addition to assessing the combination of fiscal policies between 1981 and 1986 from the perspective of generational budgets, one can simulate their joint impact. To analyze the 1981-1986 Reagan policy, I have used a very mainstream 55-period dynamic life cycle simulation model (8). This model, which incorporates quite conservative parameter estimates, predicts a small amount of crowding in from the Reagan fiscal policy prior to 1986. This may seem like a remarkable conclusion in light of the public discussion about U.S. fiscal policy, but in that discussion major features of the policy were ignored, including the change in Social Security and the short-lived changes in investment incentives.

The conclusion may also seem surprising given the popular belief that the U.S. saving rate declined dramatically in the first half of this decade. This belief is based on the Commerce Department's National Income and Product Account (NIPA) data. The NIPA data, however, improperly measure the private consumption arising from the purchase of consumer durables and count government investment, such as the construction of a bridge, as current consumption. Once one corrects the NIPA data for these mismeasurements, there is no evidence of a drop in the U.S. saving rate in the 1980s compared to the 1970s (9).

## The Danger of Misreading Fiscal Policy

Deficit delusion can have major costs. In misreading our fiscal policy we may think we are tightening up when we are doing the opposite. Having, albeit unwittingly, produced a reasonably tight fiscal policy in its first term, the Reagan Administration shifted to a looser fiscal policy in 1986 by adopting the "revenue neutral" Tax Reform Act of 1986. Although this policy left unchanged our entirely arbitrary official deficit, it certainly was not generationally neutral. By greatly reducing investment incentives, the Tax Reform Act redistributed toward older generations and loosened up fiscal policy at a time when the Reagan Administration and Congress thought that it needed to tighten up fiscal policy.

The reverse misreading of fiscal policy is likely to occur toward the end of this century. In the late 1990s there will be very substantial "surpluses" arising from a large inflow of social security "taxes." These surpluses need to be saved to help finance the benefits of the baby boom generation. Although generational budgets will show no change in the fiscal policy, those who read the stance of fiscal policy by looking at the official deficit will see large surpluses and likely advocate a loosening of policy at a time when the policy is not particularly tight.

Misreading fiscal policy may also alter expectations and affect markets that depend greatly on expectations. Unwarranted concern about U.S. fiscal policy may have been the cause of the panic that led to last October's stock market crash. Note, by the way, that the crash and the decline in the market in the 2 months prior to the crash

effectively redistributed close to \$1 trillion from older generations, who held most of the stock market, to younger and future generations, who can now purchase the same physical assets at a cheaper price. Because of our failure to assess fiscal policy and economic events in generational terms, we have still to understand that the decline in the stock market was an economic event that was the equivalent of running an extremely fiscally conservative policy. The stock market, in a matter of a few days, effectively did what many had been clamoring after for 8 years. Instead of understanding that more fiscal conservatism is now unwarranted, the general perception is that it is time to really tighten up.

## Conclusion

The development of present value generational budget accounts would greatly improve our description and analysis of fiscal policy. They would be invariant to accounting conventions, capture all fiscal policies, and show at a glance how much each generation is paying to finance the government's consumption. Such accounts would, of course, be sensitive to the choice of interest rates and to projections of future fiscal policy and future economic performance. Whether these accounts will provide more than a very rough road map remains to be seen. But even a rough road map of actual fiscal policy would be preferable to the quite precise road map of accounting whims that constitutes current descriptions of fiscal events.

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2. F. Modigliani and R. Brumberg, in *Post-Keynesian Economics*, K. Kurihara, Ed. (Rutgers Univ. Press, Rutgers, NJ, 1954).
3. See *The Economic Report of the President* (Government Printing Office, Washington, DC, 1982), chap. 4.
4. For a discussion of the literature on liquidity constraints and consumption see L. J. Kotlikoff, *What Determines Savings?* (MIT Press, Cambridge, MA, 1988).
5. This statement does not take into account policy-induced changes in the time paths of wages and interest rates that will differently affect the consumption possibilities of successive generations. A full description of the policy requires specifying these general equilibrium changes in factor prices and how they affect successive generations.
6. In the sample presented here, the government's consumption equals zero. Note that the sum of the accounts in the model presented here discounted to time  $t$  is

$$\begin{aligned}
 -H + \sum_{j=1}^{\infty} H r_{t+j} \prod_{i=1}^j (1+r_{t+i})^{-1} &= -H + \sum_{j=1}^{\infty} H \prod_{i=1}^{j-1} (1+r_{t+i})^{-1} \\
 &\quad - \sum_{j=1}^{\infty} H \prod_{i=1}^j (1+r_{t+i})^{-1} = 0
 \end{aligned}$$

7. A. J. Auerbach and L. J. Kotlikoff, *Dynamic Fiscal Policy* (Cambridge Univ. Press, Cambridge, MA, 1987), chap. 9.
8. This model is developed and analyzed by Auerbach and Kotlikoff (7).
9. See M. J. Boskin, M. S. Robinson, A. M. Huber, "Government saving, capital formation, and wealth in the United States, 1947-1985" (National Bureau of Economic Research working paper no. 2352, Cambridge, MA, August 1987), table 8, p. 26.