

# TAX INCIDENCE IN A LIFE CYCLE MODEL WITH VARIABLE LABOR SUPPLY

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## I. INTRODUCTION

Recent contributions to the theory of taxation have stressed the importance of considering the long-run incidence of factor taxation within a growth-theoretic framework.<sup>1</sup> This literature has demonstrated that the long-run incidence of a factor tax may differ substantially from the short-run incidence; indeed a factor tax that is unshifted in the short run may be shifted by more than 100 percent in the long run. An interesting corollary to these findings is that the factors which determine short-run incidence may be irrelevant for long-run incidence. For example, Feldstein [1974a] presents a growth model in which the long-run incidence of a factor tax on capital or labor is independent of the elasticity of labor supply. However, with the exception of Diamond [1970], these contributions have relied on ad hoc formulations of the determinants of individual saving and labor supply decisions. The life cycle determination of savings and labor supply and the interdependent responses of these choices to taxation have not been addressed.<sup>2</sup>

This paper extends the analysis of long-run tax incidence by considering the burden of the capital and labor income taxes in a model where both human and physical capital accumulation as well as labor supply are determined by lifetime utility maximization. We thus extend Feldstein's and Diamond's analyses of long-run tax incidence by incorporating variation in the labor supply response to taxation over the life cycle as well as the choice and timing of human capital accumulation. We show that the result on the irrelevance of the labor supply elasticity to long-run incidence requires an identical labor supply response at all ages. In the empirically plausible case that factor taxation affects labor supply differentially over the life cycle, the "elasticity of labor supply" again emerges as an important determinant of long-run tax incidence.

1. See, for example, Feldstein [1974a, 1976a and b], Diamond [1970].

2. Typically, Kaldor's assumption of a fixed savings rate out of each source of income has been made. We find this assumption implausible; if workers save any of their income, they must accumulate some capital. It is then hard to see why the same individual would save at different rates out of different income sources.

Traditional incidence models have treated labor supply as unidimensional; workers supply one quality of labor for differing numbers of hours. Labor supply is, however, a far more complicated phenomenon. Workers choose the proportion of their lifetime devoted to work, schooling, and on-the-job training; they choose how much effort to put forth, where to locate, how hard and long to search for alternative employment opportunities, as well as how many hours to work in a given time period. These other choices that workers make may have important implications for tax incidence. For example, the "Extended Life Cycle Model" of consumption described by Feldstein [1976a and b] and Kotlikoff [1979a] suggests that retirement age has a significant effect on lifetime savings patterns. In the "Extended Life-Cycle" formulation, the savings of the young respond to the expected length of the retirement period. Hence the long-run supply of capital depends on the supply of labor. Taxes that alter the age of retirement will affect the steady-state per capita levels of both capital and labor, so ultimate incidence will depend critically on the elasticity of this type of labor supply response. Econometric evidence indicates that the retirement decision may be quite sensitive to economic variables.<sup>3</sup> A wage tax that encourages retirement and raises saving will, therefore, be shifted at least partially onto capital by increasing the steady-state capital-labor ratio.

Human capital accumulation is another important dimension of labor supply choice that has received little attention in studies of the effects of taxation.<sup>4</sup> The omission of human capital may be of substantial import. Recent estimates place the human capital stock of the United States at \$997 billion compared to a business capital stock of \$1090 billion.<sup>5</sup> Heckman [1976] has shown that tax effects on human capital accumulation may differ greatly from those on either hours worked or physical capital accumulation. In particular, Heckman demonstrates that an interest income tax will actually promote human capital accumulation, while it reduces incentives for physical capital accumulation. However, he does not consider the effects of human capital taxation within a general equilibrium framework.

Section II outlines the basic model that generalizes the overlapping generations model of Samuelson [1958] and Diamond [1970] to allow for human capital accumulation and variable second-period

3. For a survey of the growing evidence on the determinants of retirement behavior, see Campbell and Campbell [1976], and Pellechio [1978].

4. The principal exception is Heckman [1976].

5. Freeman [1977]. This comparison understates the importance of human capital, as all human capital derived from nonschool sources is ignored.

labor supply. In Section III we consider the incidence of wage and capital taxes in turn. Section IV summarizes our findings and suggests some implications. An Appendix on utility maximization concludes the paper.

## II. THE MODEL

In this section we first consider the individual optimization problem and then turn to the calculation of the general equilibrium steady-state properties of the model. Individuals live for two periods, consuming goods and leisure in period 2, and goods only in period 1. During the first period, an individual's time is divided between training and work. Training occurs only in the first period; the second period is devoted to work and leisure. We assume no first-period leisure in order to highlight the importance for tax incidence of the timing of the labor supply responses.<sup>6</sup> As we demonstrate below, the inclusion of first-period leisure would not alter the conclusions provided that the compensated elasticity of labor supply when old exceeds that when young.

First-period consumption  $C_1$ , second-period consumption  $C_2$ , and second-period leisure  $l$ , are chosen to maximize an intertemporal utility function,

$$(1) \quad U(C_1, C_2, l) \quad \text{subject to} \quad C_1 + \frac{C_2}{1 + r_n} = E,$$

where  $E$  is the present value of an individual's lifetime earnings and  $r_n$  is the net rate of interest.

The optimization problem can be divided into two parts. First, given  $l$ , the individual chooses an optimal proportion  $S$  of the first period for training in order to maximize  $E$ , where

$$(2) \quad E = W_n(1 - S)H_0 + W_nH(S)(1 - l)/(1 + r_n).$$

Here  $W_n$  is the net wage rate,  $H_0$  is each individual's initial endowment of human capital, and  $H(S)$  is the human capital production function with assumed positive first and negative second derivatives. A worker with human capital  $H(S)$  is equivalent to  $H(S)/H_0$  untrained workers. For simplicity, it is assumed that the only input into human capital formation is the worker's time. The measure of human

6. In addition, zero first-period leisure seems a reasonable assumption. Primary workers typically do not vary their participation much until retirement years. While there is some variation in the labor force, this is primarily due to differing training choices.

capital accumulation  $S$  can be thought of as either the proportion of the first period spent in school or the proportion of time devoted to on-the-job training. Maximization of  $E$  for given  $l$ ,  $W_n$ , and  $r_n$  yields the first-order condition (3) and the demand for training relationship (4):

$$(3) \quad H_0 = H'(S)(1 - l)/(1 + r_n)$$

$$(4) \quad S = S(l, r_n).$$

Differentiating (4) implies that<sup>7</sup>

$$(5) \quad \frac{dS}{dr_n} < 0, \quad \frac{dS}{dl} < 0, \quad \frac{dS}{dW_n} = 0.$$

Reducing second-period leisure increases the amortization period for human capital investment and leads to more training. Similarly, a fall in the interest rate raises the present value of the return and increases the optimal level of human capital accumulation. Since a change in the net wage varies both the costs and the return on human capital investment in equal proportion, it has no effect on the optimal level of training. Substituting for  $C_2$  from (1), (2), and (4), the consumer maximizes (6) over  $C_1$  and  $l$ :

$$(6) \quad U(C_1, W_n[(1 - S)H_0(1 + r_n) + H(S)(1 - l)] - C_1(1 + r_n), l).$$

First-order conditions are

$$U_1 = (1 + r_n)U_2$$

$$U_3 = W_n H(S)U_2.$$

Evaluation of tax incidence requires a standard of comparison. In what follows, we consider the incidence of compensated tax changes. This is equivalent to assuming that the government spends tax revenues by giving lump-sum rebates. The same results hold if we impose the requirement that government spend (or save) the revenues in exactly the way the taxpayers would have. In the Appendix we derive formulae for changes in  $C_1$ ,  $C_2$ , and  $l$  arising from compensated changes in the tax rates  $t_w$  and  $t_r$ . The compensation takes the form of a lump-sum rebate of tax proceeds in the same period as the tax is incurred. Under the assumptions outlined in the

7. These results depend on the assumption that time is the only input in the production of human capital. If goods as well as time are inputs, the effects on  $S$  of changes in  $r$ , and  $l$  remain the same. Increases in  $W$  will lead to higher levels of  $s$ , however.

Appendix, we record the compensated tax change derivatives in (7) where  $c$  stands for compensated:<sup>8</sup>

$$(7) \quad \frac{dC_1}{dt_w^c} < 0, \quad \frac{dl}{dt_w^c} > 0, \quad \frac{dC_1}{dt_r^c} > 0, \quad \frac{dl}{dt_r^c} < 0.$$

A labor income tax when compensated increases leisure in the second period while lowering consumption in both periods. Since savings are given by  $Y_1 - C_1$ , where  $Y_1$  is earnings in the first period, it follows from (7) above that a labor income tax raises savings. The effects of an interest income tax are more complex. This tax increases human capital accumulation, making second-period leisure less attractive. The resulting increase in  $E$  leads to increases in both  $C_1$  and  $C_2$ . The compensated interest income tax reduces saving both because  $C_1$  rises and  $Y_1$  falls due to the increase in  $S$ .

At the macro level, the physical capital stock is determined by the savings of the young. Letting  $g$  be the population growth rate, we may write physical capital per young person  $k$ , as

$$(8) \quad k = [W_n(1 - S)H_0 - C_1 + T_1]/(1 + g),$$

where  $T_1$  is first-period lump-sum rebates of taxes. Human capital per young person  $h$  is

$$(9) \quad h = (1 - S)H_0 + H(S)(1 - l)/(1 + g).$$

Combining (8) and (9) the economy's steady state capital-“effective labor” ratio  $k^*$  is simply

$$(10) \quad k^* = \frac{k}{h} = \frac{W(1 - S)H_0 - C_1}{(1 - S)H_0(1 + g) + H(S)(1 - l)}.$$

In (10)  $W$  is the gross wage rate, and we have made use of the fact that  $T_1 = t_w W(1 - S)H_0$ .

Expression (10) indicates that the capital-labor ratio  $k^*$  is determined by individual choice of  $C_1$ ,  $C_2$ ,  $l$ ,  $S$  for given gross factor returns and tax parameters. We assume a constant returns production technology  $y = f(k)$ , with factor price frontier:

$$(11) \quad W = \phi(r), \quad \phi' = -k^*.$$

Steady-state equilibrium requires that the gross factor returns  $W$  and

8. Letting  $Z$  stand for  $C_1$ ,  $C_2$ , or  $l$ , we see that

$$\frac{dZ}{dt^c} = \frac{\partial Z}{\partial t} + \frac{\partial Z}{\partial T_1} \frac{\partial T_1}{\partial t} + \frac{\partial Z}{\partial T_2} \frac{\partial T_2}{\partial t},$$

where  $t$  represents  $t_w$  or  $t_r$  and  $T_1$  and  $T_2$  represent first- and second-period compensation.

$r$  determining  $k^*$  be consistent with the  $W$  and  $r$  dictated by the production function at  $k^*$ . Thus, we close the model with the condition,

$$(12) \quad k^*(r, W) = \frac{W(1 - S(l, r_n))H_0 - C_1(r_n, W_n)}{(1 + g)(1 - S(l, r_n))H_0 + H(S)(1 - l(r_n, W_n))}.$$

We assume below that steady-state solutions are unique and stable.

### III. TAX INCIDENCE

Before considering the incidence of a wage tax in our model, it is useful to describe the special case considered by Feldstein. His result on the irrelevance of labor supply elasticity to long-run incidence is immediate with a slight modification of (10). In the absence of human capital formation and allowing leisure in both periods,  $k^*$  is

$$(13) \quad k^* = (WL_1 - C_1)/[L_1(1 + g) + L_2],$$

where  $L_1$  and  $L_2$  are period 1 and 2 supplies of labor, respectively. Assuming that indifference curves are homothetic, so that the savings rate is independent of the wage, we can write

$$(14) \quad C_1 = z(r_n) \left( W_n L_1 + \frac{W_n L_2}{1 + r_n} + T_1 + \frac{T_2}{1 + r_n} \right),$$

where

$$(15) \quad \begin{aligned} T_1 &= t_w WL, \\ T_2 &= t_w WL_2 + (WL_1 - C_1)t_r \cdot r/(1 + r_n). \end{aligned}$$

Using (15), we may rewrite the expression for  $C_1$  as

$$(16) \quad C_1 = \frac{z(r_n)(WL_1(1 + t_r r/(1 + r_n)) + WL_2/(1 + r_n))}{1 + z(r_n)t_r r/(1 + r_n)}.$$

Examination of (13) and (16) clearly shows that any compensated tax change which alters  $L_1$  and  $L_2$  by the same proportion will change  $C_1$  by the same proportion leaving  $k^*$  in (13) unaltered. Under this assumption of equal labor supply elasticities, the long-run burden of a compensated wage tax will fall entirely on labor, since gross factor returns are dictated by the production function and  $k^*$  is unchanged. An interest income tax will, on the other hand, change gross returns, but any induced equiproportionate changes in  $L_1$  and  $L_2$  will have no impact on ultimate incidence. Thus, an interest income tax may be

shifted, but labor supply factors play no role.

From the preceding discussion, it should be clear that the irrelevance of labor supply for long-run tax incidence requires very special assumptions. Given the ubiquitous forty-hour work week, on the one hand, and the substantial variation in individual working lives, on the other, the case for disproportionate labor supply response over the life cycle seems compelling. We therefore consider wage tax incidence in the general model we have outlined.

While our model assumes no first-period leisure, this feature of the model is not critical to the qualitative results. Equations (13) and (16) imply that if  $L_2$  falls by proportionately more than  $L_1$ , the capital-labor ratio will rise, shifting the tax burden onto capital. Similarly, if labor supply is more responsive to taxation when young than when old, the net impact of the labor supply response is to shift more of the tax burden onto labor. The qualitative results we describe below remain true even with variable first-period leisure as long as the compensated elasticity of labor supply of the old exceeds that of the young.

The requirement of equal compensated labor supply elasticities at each age is fairly stringent and requires more than simply homotheticity of the utility function. For example, consider the utility function,

$$U = \log C_1 + \frac{\log C_2}{1 + \rho} + \log l_1 + \frac{\log l_2}{1 + \rho},$$

where  $l_1$  and  $l_2$  are first- and second-period leisure, respectively, and  $\rho$  is the rate of time preference. While this utility function exhibits completely inelastic uncompensated labor supply curves, the compensated labor supplies are elastic. Indeed, the compensated elasticity of labor supply when old will exceed that when young provided that  $r > \rho$ ; for  $r = \rho$  the two are equal, and for  $r < \rho$  the relative magnitudes reverse. The condition  $r > \rho$  entails increasing consumption of leisure as one ages, presumably what we observe in the real world.<sup>9</sup> Even assuming identical compensated supply elasticities at all ages, we see that our model is useful for thinking about the incidence of a pro-

9. When one adds human capital to the analysis, the condition of equal proportionate reduction in labor supply is less likely to hold. In the case with no human capital accumulation, equal compensated elasticities of leisure lead to an unequal elasticity of labor supply if the initial level of leisure when old exceeded that when young. Adding human capital makes the elasticity of labor supply when young even smaller, since the reduction in labor supply when old leads to a shorter training period when young and hence more labor supply when young; i.e., even for equal percentage reductions in leisure in the two periods, there is a substitution of labor for training when young due to the shorter amortization period.

gressive labor income tax. Since the rate of taxation of human capital would be higher when old, the percentage reduction in labor supply when old would exceed the reduction when young, yielding the same type of shifting we now investigate.

In order to find the incidence of a wage tax, we totally differentiate the steady-state condition (12). This yields

$$(17) \quad \frac{dw}{dt_w^c} = -k^* \left[ \left( -WH_0 + k^*(H_0(1+g) - H'(1-l)) \right) \frac{dS}{dl} + k^*H \right] \frac{dl}{dt_w^c} + \frac{dC_1}{dt_w^c} \Big/ \frac{h}{f_{11}} - m_1 + m_2 + k^*m_3,$$

where

$$m_1 = \phi'H_0(1-S) - WH_0 \left( \frac{ds}{dr} + \frac{ds}{dl} Q_l \right)$$

$$m_2 = \frac{dC_1}{dW} \phi' + \frac{dC_1}{dr} \quad Q_l = \frac{dl}{dW} \phi' + \frac{dl}{dr}$$

$$m_3 = [H'(1-l) - H_0(1+g)] \left( \frac{dS}{dr} + \frac{dS}{dl} Q_l \right) - HQ_l.$$

We evaluate the derivative at  $t_w = 0$ , that is, at the no-tax equilibrium. Given the assumption that substitution effects dominate,  $dC_1/dW > 0$ , and  $dC_1/dr < 0$ . Hence all terms in the denominator of (17) are unambiguously negative except for  $m_1$ . Stability requires that the denominator be negative, otherwise the wage would rise or fall without bound if perturbed away from the equilibrium level. We therefore assume that  $m_1$  is small enough to insure stability.

Turning to the numerator of (17), we first consider the case of no human capital accumulation ( $S = 0$ ,  $ds/dl = 0$ ). In this case the numerator is

$$(18) \quad -k^*2H \frac{dl}{dt_w^c} + k^* \frac{dC_1}{dt_w^c} < 0$$

with the inequality following from (7). Hence a labor income tax unambiguously raises the gross of tax wage. The economic mechanism is simple; earlier retirement raises the savings rate, increasing capital intensity. In addition, it reduces the steady-state per capita labor supply. These effects may increase the capital-labor ratio by enough to leave a higher net of tax wage in the post-tax equilibrium. The greater the retirement elasticity and the smaller the elasticity of substitution in production, the greater is the shifting of the wage tax onto capital.

The second special case we consider is that of fixed leisure (retirement) and variable human capital accumulation. In this case labor bears exactly 100 percent of the tax; the numerator of (17) vanishes except for  $dC_1/dt_w$ . However, since  $l$  is fixed and  $dS/dt_w = 0$ , lifetime earnings are fixed and  $dC_1/dt_w^c = 0$ . Thus, human capital bears the entire burden of a wage tax, despite being variable. This result therefore extends Feldstein's original results on the irrelevance of variable labor supply to another form of labor supply; i.e., elasticity of the human capital supply function is not a sufficient condition for shifting of a labor income tax.<sup>10</sup>

When both  $S$  and  $l$  are free to vary, labor will always shift some part of the tax as long as  $r$  exceeds  $g$  in the initial equilibrium. From (3) it follows that

$$(19) \quad H_0(1 + g) - H'(1 - l) < 0 \quad \text{if} \quad r > g,$$

Now,  $dS/dl < 0$ , so the entire numerator is unambiguously negative. Hence, part of the tax will always be shifted in the "normal" case where no "capital glut" exists. The economic logic is clear. The reduction in second-period labor supply leads to a decrease in first-period consumption raising  $k$ . This effect is magnified by the increase in period 1 earnings caused by the reduction in training. In addition, when  $r > g$ , there is a decline in the steady-state labor force per young worker  $h$  because of the shorter training period  $s$ . We see that in a life cycle model, accumulation of human capital indirectly affects labor tax incidence when the length of the amortization period changes.

Could these effects be significant? There is little empirical evidence on taxation's effect on human capital accumulation, so not much can be said about training effects. The evidence regarding retirement effects is mixed. At the macro level there is certainly no simple correlation between the historic aggregate savings rate and the age of retirement. Between 1900 and 1971 the participation rate of men over sixty-five fell from 63.1 percent to 25.5 percent.<sup>11</sup> At the same time the U. S. aggregate savings rate remained roughly constant, averaging 17.7 percent between 1898 and 1916 and 15.8 percent between 1949 and 1969.<sup>12</sup> Of course, many other things were changing during this period. Most notably, social security was introduced,

10. It should be emphasized that this result depends on the assumption that time is the only input into human capital production. If goods also enter the human capital production function,  $dS/dt_w$  is positive, assuming that the expenditure on goods is not expensed.

11. Munnell [1974].

12. David and Scadding [1974].

possibly offsetting any effects of increased retirement.<sup>13</sup> Feldstein's [1976b] international regression analysis of savings rates finds a substantial effect of retirement age. The estimated coefficients imply that a 20 percent decrease in the proportion of those working over age sixty-five would lead to a 40 percent increase in the savings rate. In the long run this would lead to an equal change in the capital-labor ratio. At the micro level, Kotlikoff [1979b] regresses net worth accumulation against variables, including the expected age of retirement, and finds that an additional three years of retirement raises net worth accumulation by close to 10 percent. He suggests that this is likely to be a lower bound. Significant shifting is implied by these estimates. With a Cobb-Douglas technology, a 20 percent change in the capital-labor ratio leads to about a 15 percent change in the interest rate and a 5 percent change in the wage rate. Hence, if a 25 percent wage tax leads to a 20 percent decrease in the number of economically active over sixty-five, about 20 percent of the burden will be shifted by labor even ignoring human capital considerations. With a less elastic production function even more of the tax will be shifted.

The incidence of an interest income tax can also be studied in our model. The results generalize those of Diamond [1970] by allowing for life-cycle labor supply as well as consumption decisions. The procedure is the same as in the wage tax case. Differentiating the steady-state condition yields

$$(20) \quad \frac{dr}{dt_r^c} = \left[ -m_4 \frac{dS}{dl} + k^*H \right] \frac{dl}{dt_r^c} - \frac{dC_1}{dt_r^c} - m_4 \frac{dS}{dt_r^c} / \frac{h}{f_{11}} - m_1 + m_2 + k^*m_3,$$

where

$$(21) \quad m_4 = [wH_0 - k^*(H_0(1+g) - (1-l)H')] > 0$$

and is unambiguously positive as long as  $r > g$ . The denominator of (20) is the same as in the wage tax case.

Again, we first consider the case without human capital accumulation:

$$(22) \quad \frac{dr}{dt_r^c} = k^*H \frac{dl}{dt_r^c} - \frac{dC_1}{dt_r^c} / \frac{h}{f_{11}} - m_1 + m_2 + k^*m_3.$$

13. See Feldstein [1976a]; Munnell presents time series evidence that holding social security constant, early retirement alone increases the savings rate.

From the restrictions found in (7) it is easy to see that the gross interest rate is increased by a capital income tax. It is even possible for the net interest rate to rise. In addition to reducing savings by encouraging first-period consumption for a fixed earnings stream, the tax encourages second-period work effort. If we append the additional condition that  $dl/dt_r^c = 0$ , our model reduces to Diamond's [1970] model in which the same results hold.

Second, if  $l$  is fixed, but  $S$  is free to vary, and  $r > g$ , some of the tax burden will be borne by labor. In this case we have

$$\frac{dr}{dt_r^c} = -\frac{dC_1}{dt_r^c} - m_4 \frac{dS}{dt_r^c} / \frac{h}{f_{11}} - m_1 + m_2 + k^*m_3.$$

Thus, unlike the wage tax, the incidence of an interest income tax is affected directly by the elasticity of human capital supply. The more sensitive is human capital accumulation to changes in the net interest rate, the greater the shifting of the tax. This shifting arises, in part, from an increase in the steady-state human capital stock when  $r > g$  and, in part, from a decline in earnings and thus savings when young. The implication of these results is that capital need not bear an interest income tax even in the long run. Even if, as is frequently assumed, consumption is completely insensitive to the interest rate, part of the burden of the interest tax will be shifted to labor through human capital variation. Unfortunately, little is known about the dependence of human capital investment on interest rates.

#### IV. CONCLUSION

This paper calls attention to the economic implications for long-run tax incidence of viewing labor supply as multifaceted. We have demonstrated that allowing for variable retirement and human capital accumulation alters conventional conclusions regarding long-run tax incidence. In particular, the responsiveness of retirement age and training duration to the taxation of wage income plays an important role in shifting a wage tax onto capital. Second, the interest income tax may be shifted onto labor even if consumption is unresponsive to the interest rate through induced changes in human capital accumulation. Only more realistic models and empirical estimates of key parameters will permit evaluation of the importance of these effects.

The analysis has obvious relevance to the design and study of

social security. Social security represents an example of differential taxation of labor income in different periods. Hence the effects described here are likely to be magnified. The results suggest that the heavy taxation of post-retirement benefits may have at least one virtue. The induced early retirement is likely to raise the equilibrium real wage. It would be fruitful to generalize the analysis to consider the effect of allowing different tax rates on laborers at different wages; such an analysis would shed light on the consequences of a progressive tax system with only very limited income averaging. In addition, extending the model to allow for less than perfect substitutability between skilled and nonskilled workers would permit more realistic modeling of the general equilibrium distributional effects of human capital taxation.

#### APPENDIX: UTILITY MAXIMIZATION

The first-order expressions given in the text are

$$(A.1) \quad \begin{aligned} L_1 &\equiv U_1 - (1 + r(1 - t_r))U_2 \\ L_2 &\equiv U_3 - (1 - t_w)WH(S)U_2. \end{aligned}$$

Letting  $t^c$  stand for either of the two compensated taxes,  $t_w^c$  and  $t_r^c$ , and  $T_1$  and  $T_2$  for compensation periods one and two, respectively, we totally differentiate (A.1) with respect to  $C_1$  and  $l$  to obtain (1 and 2 stand for derivatives with respect to  $C_1$  and  $l$  when associated with  $L$  and  $C_1$  and  $C_2$  when associated with  $U$ ):

$$(A.2) \quad \begin{pmatrix} L_{11} & L_{12} \\ L_{21} & L_{22} \end{pmatrix} \begin{bmatrix} \frac{dC_1}{dt^c} \\ \frac{dl}{-dt^c} \end{bmatrix} = \begin{bmatrix} -\frac{\partial L_1}{\partial t} - \frac{\partial L_1}{\partial T_1} \frac{\partial T_1}{\partial t} - \frac{\partial L_1}{\partial T_2} \frac{\partial T_2}{\partial t} \\ -\frac{\partial L_2}{\partial t} - \frac{\partial L_2}{\partial T_1} \frac{\partial T_1}{\partial t} - \frac{\partial L_2}{\partial T_2} \frac{\partial T_2}{\partial t} \end{bmatrix},$$

where

$$(A.3) \quad \begin{aligned} L_{11} &= U_{11} - 2U_{12}(1 + r_n) + U_{22}(1 + r_n)^2 \\ L_{12} &= -U_{12}W_nH + U_{13} + U_{22}W_nH(1 + r_n) - U_{23}(1 + r_n) \\ L_{22} &= U_{22}W_n^2H^2 - 2U_{23}W_nH + U_{33} - U_2W_n \frac{dH}{dl} \\ T_1 &= t_wW(1 - S)H_0 \\ T_2 &= t_wW(1 - l)H + (W(1 - S)H_0 - C_1)rt_r. \end{aligned}$$

From (A.2) and (A.3) it follows immediately that

$$\begin{aligned}
 \frac{dC_1}{dtw^c} &= \frac{WHL_{12}}{D} < 0 \\
 \text{(A.4)} \quad \frac{dl}{dtw^c} &= \frac{-U_2WHL_{11}}{D} > 0. \\
 \frac{dC_1}{dtr^c} &= -L_{22}U_2r - L_{21}U_2W \frac{\partial H}{\partial tr} / D > 0 \\
 \frac{dl}{dtr^c} &= L_{12}U_2r + L_{11}U_2W \frac{dH}{dtr} / D < 0.
 \end{aligned}$$

We evaluated the above expressions at initial taxes of zero. Assuming that conditions for a maximum are satisfied,  $L_{11} < 0$ ,  $L_{22} < 0$ , and  $D = L_{11}L_{12} - L_{12}^2 > 0$ . In (A.4)  $dl/dtw^c$  is unambiguously negative, a compensated wage tax induces greater second period leisure. The sign of  $dC_1/dtw^c$  depends on the sign of  $L_{12}$ . If we assume that  $U_{12} = U_{13} = 0$  and  $U_{23} \geq 0$ , then  $L_{12}$  is negative as is  $dC_1/dtw^c$ . These assumptions are in accord with the standard separable intertemporal utility functions used in growth models of this kind. The signs of  $dC_1/dtr^c$  and  $dl/dtr^c$  are also unambiguous, assuming that  $L_{12} < 0$ . The sign of  $dC_1/\partial tr$  reflects the substitution away from the now more expensive second-period consumption as well as the increase in consumption in both periods due to the greater supply of labor when old. The negative sign of  $dl/dtr^c$  results from the higher price of leisure induced by greater human capital accumulation. In addition, the price of second-period leisure rises relative to first-period consumption and means a substitution away from leisure toward first-period consumption.

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