SOCIAL SECURITY AND EQUILIBRIUM CAPITAL INTENSITY*

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The past few years have witnessed a growing concern over an aggregate capital shortage. The debate has identified the unfunded social security system as a major factor in reducing aggregate savings and the capital stock since its introduction in 1935. The unfunded financing of the social insurance program is central to the discussion. While the original 1935 legislation authorized the accumulation of a large trust fund, this goal was essentially abandoned with the 1939 amendments. These and other amendments severely weakened the link between taxes paid in and benefits received, permitting, it is argued, a time path of aggregate consumption in excess of what would have occurred in the absence of social security.

Recent empirical work at the macro level suggests a substantial reduction in the capital stock generated by social security. One initial estimate put forth by Martin Feldstein [1974] entails a 38 percent decrease in the capital stock in the long run. Obviously a 38 percent reduction in the capital stock has enormous implications for the steady state level of welfare and factor rewards. This paper investigates the impact of social security on the steady state capital stock of an idealized economy. The purpose of this analysis is to determine whether a simplified economy would exhibit reductions in the capital stock of the 40 percent order of magnitude. The model permits a comparison of general equilibrium steady state effects in which factor returns respond to the fall in the capital stock with the partial equilibrium reduction in the capital stock when factor rewards are held constant. Our analysis considers growth in population and productivity and permits the age of retirement to respond to social security. As Feldstein and others have pointed out, social security may induce

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early retirement due to an implicit tax on post-sixty-five earnings.[1] The savings response of the young to a planned earlier retirement is likely to be positive; hence, earlier retirement may lead to more aggregate savings and thus dampen the reduction in the capital stock due to the introduction of an unfunded social security system. We are particularly interested here in determining whether more savings induced by reasonable retirement responses could substantially offset or even reverse social security's otherwise potential reduction in the capital stock.

The paper is organized in the following manner. The first section reviews the theory of social security and life cycle savings. We analyze here the "one for one" replacement of private savings by social security taxes, the retirement effect, and the effect of changes in lifetime wealth due to the yield of the social security system. In sections II through IV we present our comparative static steady state results for an economy characterized by life cycle savers. Calculations are presented for a range of initial growth rates, interest rates, and retirement ages. The replacement, retirement, and yield effects of unfunded social security on the steady state capital stock are each analyzed in turn. Section V considers some issues not captured in our life cycle model and raises some doubts at the macro level about the extent of social security's reduction in the capital stock. Section VI summarizes the findings of this paper.

I. The Theory of Social Security and Life Cycle Savings[2]

The different effects of social security on life cycle savings are easily understood with the help of Figure I. In the diagram a representative life cycler with fixed life span D faces, in the absence of social security, an earnings stream such as $W(t)$ and chooses a consumption stream such as $C(t)$ and an age of retirement R. The choice of consumption at every age as well as the age of retirement arises from the maximization of an intertemporal utility function subject to the lifetime budget constraint (assuming no bequests) LTW:

$$\int_0^D C(t)e^{-rt}dt = LTW = \int_0^R W(t)e^{-rt}dt, \tag{1}$$

where r = the rate of interest.

At the micro level the introduction of social security reduces the earnings profile by the amount of the social security tax $\theta \cdot W(t)$ prior to retirement and provides a social security benefit stream $B(t)$

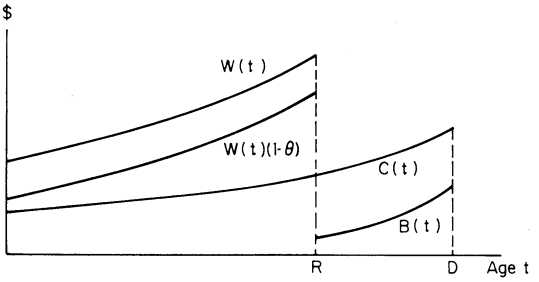

FIGURE I

thereafter. The new budget constraint facing the individual is

$$(2) \qquad LTW' = \int_0^{R'} W(t)(1-\theta)e^{-rt}dt + \int_{R'}^{D} B(t)e^{-rt}dt.$$

If retirement does not change ($R' = R$), and the social security system offers an implicit yield on paid-in taxes equal to the market rate of interest r, then lifetime wealth is not affected ($LTW' = LTW$) by the social security system, and the consumption profile is unaltered. Under these assumptions, social security taxes simply replace private savings out of earnings one dollar for one dollar prior to retirement age R. This will be referred to below as the replacement effect.

The assumption of a social security yield equal to the rate of interest requires that

$$(3) \qquad \int_0^{R} W(t)\theta e^{-rt}dt = \int_R^{D} B(t)e^{-rt}dt.$$

Equation (3) states that the present value of lifetime taxes paid in must equal the present value of lifetime benefits received for the system to be actuarially fair. Assuming a stable population with growth rate n and identical individuals, we can write the social security budget constraint as

$$(4) \qquad B(t) = \theta W(t)L/M,$$

where L stands for the labor force and M for the number of retirees. Equation (4) simply equates benefits received to taxes paid in. Using the assumption of a stable population, L/M we may express as

$$(5) \qquad \frac{L}{M} = \frac{\int_0^{R} e^{-na}da}{\int_R^{D} e^{-na}da}.$$

In (5) we integrate over age cohorts. Letting real wages grow at a rate g and combining (3), (4), and (5), we see that when $r = n + g$, social security yields the market rate of interest, and there is no change in lifetime wealth from the yield on social security taxes.

A lifetime wealth increment (the yield effect) arising from growth rates of population plus productivity in excess of the market interest rate would mean a higher LTW and an upward shift in the consumption profile.[3] This lifetime wealth increment would reduce savings at every age implying a greater than one for one reduction in private savings. The yield effect would, correspondingly, be opposite in sign if the interest rate exceeded the growth rates of population plus productivity. A reduction in the age of retirement (the retirement effect) will reduce lifetime wealth by shortening the earnings stream. This will shift the consumption profile downward increasing savings at young ages.[4]

So far the analysis has been partial equilibrium; we have not yet taken into account the general equilibrium shifts in $W(t)\cdot(1-\theta)$ and $C(t, LTW)$ arising from changes in the steady state capital stock and age of retirement. A reduction in the steady state capital stock may be associated with higher or lower wage earnings and interest rates due to the simultaneous reduction in labor input arising from earlier retirement. In the most likely case where the wage falls and the interest rate rises, both $W(t)(1-\theta)$ and $C(t, LTW)$ will shift downward. LTW will decline not only because earnings at every age are lower, but also because the interest rate at which future earnings are discounted is higher. The shape of the consumption profile is also likely to change as the discrepancy between the rate of time preference and the interest rate changes.

Each one of the three effects mentioned above—replacement, yield, and retirement—plays a role in determining the final steady state capital stock; the three effects are examined within the life cycle model of Section II. While social security's wealth replacement and lifetime yield have unambiguous impacts on the steady state capital stock, the theoretical effect on the capital stock of induced early retirement is ambiguous. While it is true that the young will accumulate more, the earlier retirement age may leave older cohorts with less accumulated wealth at a given age. Since the steady state capital stock is simply the summation over age cohorts of private wealth holdings, early retirement may increase or decrease the capital stock.

While our purpose here is to determine the quantitative importance of these three effects of social security within an idealized life cycle economy, we realize that social security may alter the capital

stock through other channels. For example, if capital markets are imperfect and one cannot borrow against future social security benefits, fixed savings goals (e.g., for a down payment on a house or for children's education) prior to retirement age may lead to a one-for-one reduction in consumption up to a certain age, and an increase in consumption compared to previous levels thereafter. In other words, the consumption profile could rotate counterclockwise at a given age. By not considering other mechanisms through which social security affects savings, we feel we have, in net, biased our results toward a larger reduction in the capital stock. We shall return to this question of alternative specifications in Section V.

II. Social Security and the Steady State Capital Stock in a Life Cycle Economy

In this section we derive the formula for the steady state capital stock in a simplified life cycle economy and consider changes in the steady state capital intensity induced by the introduction of an unfunded social security system. While there is more than one way in which a social security system can become unfunded, for the United States the unfunding resulted from paying out benefits to retirees who had paid little or nothing into the system. The scenario of a historical reduction in U.S. savings relative to consumption entails increased consumption by retirees receiving unanticipated social security benefits without a concomitant reduction in consumption by young taxpayers (the replacement effect predominates). After a transition period the economy arrives at a new steady state with a smaller capital intensity. The extent of the steady state changes will, in general equilibrium, depend on preferences. We specify below preferences exhibiting a constant elasiticity of marginal utility l and a time preference parameter ρ. While we shall consider variations in these taste parameters, we stress that our results may be sensitive to the general choice of utility function as well as the assumption of a fixed age of death. Unfortunately, more realistic considerations of family structure and increasing probabilities of death preclude obtaining differentiable formulas of the sort developed here.[5]

Consider, then, an economy characterized by identical individuals, a constant population growth rate n, constant labor augmenting productivity growth g, a fixed life span of D years, and a retirement age R. We assume an intertemporal utility function of the form,

$$U = \int_0^D e^{-\rho t} \frac{C_t^{1-l}}{1-l} dt, \tag{6}$$

where ρ is the rate of time preference and l is the elasticity of marginal utility. Letting I stand for the present value of lifetime wealth, we can write the consumption profile as

$$C(t) = \lambda I e^{(r-\rho)t/l}, \tag{7}$$

$$\lambda = \frac{(r(1-l)-\rho)/l}{e^{(r(1-l)-\rho)D/l} - 1}, \tag{8}$$

and the lifetime budget constraint is

$$I = \int_0^R W(1-\theta)e^{-(r-g)t}dt + \int_R^D Be^{-(r-g)t}dt. \tag{9}$$

In (4) W and B are, respectively, the wage and benefit prevailing at age zero and are assumed to grow at the rate g. To make our calculations as realistic as possible, we shall consider wage and interest income taxes. Hence in (4), W and r are the net (of wage tax) wage and the net interest rate. Throughout the analysis we assume no government savings; i.e., expenditures on current government consumption equal wage and interest tax payments. In addition, as the social security budget constraint dictates, all social security tax contributions are immediately paid out as benefits.

The steady state capital stock of this economy is simply the aggregate of individual private wealth holdings. To find private holdings at each age, we solve the two differential equations for workers and retirees given in (10) and (11), respectively:[6]

$$\frac{dK_L(t)}{dt} = W(1-\theta)e^{gt} - C(t,I) + rK_L(t) \tag{10}$$

$$\frac{dK_m(t)}{dt} = Be^{gt} - C(t,I) + rKm(t). \tag{11}$$

The solution of (11), using the initial condition that $K_L(0) = 0$, gives the capital stock owned by workers at each age, $K_L(t)$. Similarly, from the terminal condition $K_m(D) = 0$ and (11), we obtain an equation for the capital stock of retirees at each age, $K_m(t)$. Integrating piecewise the capital owned by workers and retirees over age cohorts gives the formula for the aggregate steady state capital stock K per age zero worker measured in efficiency units. In this formula (12) (see Appendix A), the absence of capital owned by the government incorporates our assumption of zero social security funding. The terms H_1, H_2, H_3, H_4 are defined in Appendix A:

$$K = \frac{W}{r-g}[H_1 - \theta H_2 + H_3 H_4]. \tag{12}$$

Equation (12) indicates that the steady state capital stock depends on net factor returns, growth rates, the ages of retirement and death, preference parameters, and the social security tax rate θ. Using equation (12), we now consider the effect on the capital stock of changes in θ evaluated at an initial θ equal to zero. The numbers presented below are calculated assuming a 10 percent differential change in the social security tax rate, a 30 percent wage tax (tw), and a 50 percent interest income tax (tr).

III. Partial Equilibrium Effect

In this section we consider the reduction in the stock of capital resulting from unfunded social security holding, for the movement, W and r constant. Taking W and r as fixed ignores the disequilibrium that would arise in a closed economy in the factor market for capital. The demand by firms for capital at the initial levels of W and r would exceed the now lower household supply of capital. If we also hold the age of retirement R constant and consider initial conditions given by $r = n + g$, the partial equilibrium effect of social security on the capital stock is independent of preferences. A comparison of general with partial equilibrium effects is interesting in the following sense. If a simple life cycle model indicates that the general equilibrium effects differ little from the partial equilibrium effect, one would have more confidence in using the easily calculable partial equilibrium effect as an estimate of the total effect. In addition, the partial and general equilibrium changes are identical in the case of a small open economy whose interest rate is pegged from abroad. In this case of internationally mobile capital, the introduction of social security will reduce domestic ownership of the world capital stock, but not necessarily the capital in place in the home country, since foreign owned capital can flow in.

From (12), the term $-W\theta H_2/(r - g)$ is the partial equilibrium effect of social security on K holding W, r, and R constant. Evaluating $-H_2$ under the assumption $r = n + g$,[7] we see that the partial equilibrium effect is

$$\frac{-W\theta H_2}{n} = \frac{-W\theta}{n}\left(R + (R - D)\frac{L}{M}e^{-nD}\right). \tag{13}$$

From (13) it is clear that for this case of $r = n + g$ the partial equilibrium effect is independent of tastes; by assumption nothing has happened to change lifetime wealth; hence the consumption profile remains unaltered, and the change in accumulated capital

TABLE I

PARTIAL EQUILIBRIUM CHANGE IN THE CAPITAL STOCK

$$\frac{dK}{K/W, r, R, r = n + g} = -d\theta \left[\frac{R}{L} + \left(\frac{R - D}{M}\right) e^{-nD}\right] \frac{(1 - \alpha)r}{\alpha n} \frac{(1 - tw)}{(1 - tr)}$$

		Age of death		
		50 (percent)	55 (percent)	60 (percent)
Retirement age	40	−35.90	−38.87	−41.78
	45	−36.93	−40.07	−43.06
	50	−38.03	−41.39	−44.37

The following values are assumed:
$r = 0.04$, $d\theta = 0.10$, $n = 0.02$, $tr = 0.5$, $tw = 0.3$, $\alpha = 0.3$, $g = 0.02$.

depends only on the changes in the (net of social security tax) earnings and benefit streams, i.e., the replacement effect. To obtain empirical estimates of the magnitude of this partial equilibrium change, we convert (13) into a capital stock percentage change. For the Cobb-Douglas production function, $K = W_g L/r_b\ \alpha/1 - \alpha$, where w_g and r_g are labor and capital gross factor returns, and α is capital's share. Dividing this expression into (13) yields the equation in Table I; Table I calculates partial equilibrium capital stock changes for different values of R and $D - R$.

The parameter values in Table I were chosen to approximate roughly the U. S. situation. The working period and life spans should be considered as measured from the age at which work begins. If age 20 is taken as the average age at which young people enter the labor force, a retirement age of 45 in the table corresponds to a real world retirement age of 65.

Table I reports extremely large partial equilibrium changes in the capital stock. Even if we reduce these numbers by the ratio of capital to total assets in the public's portfolio, a number about 0.8, the partial equilibrium effects are still substantial.[8]

The temptation to invoke the partial equilibrium results becomes apparent when one observes that the partial equilibrium effect reported above is equivalent to the ratio of the unfunded social security deficit to the initial capital stock (see Appendix B). The deficit of the social security system is defined as the present value difference between future benefits and future tax obligations of the current living population. In this partial equilibrium analysis, a fully funded trust fund would have assets exactly covering that differential. Hence the partial equilibrium effects of Table I are equivalent to the reduction

in the capital stock from a one time eating up of a fully funded social security trust fund.

Equation (13) indicates that for values of $R = 45$, $L = 55$, and $r - g = 0.02$ a fully funded trust fund would have assets equivalent to about thirty times annual tax contributions. In addition, an examination of (13) reveals that the fully funded trust fund is larger the longer the working period for a fixed life span. It is also larger the longer the life span for a fixed working period. A longer working period for a fixed length of life implies higher yearly benefits after retirement (holding θ fixed). The higher benefits are financed partly out of greater tax revenue due to more older workers and partly out of a larger trust fund. A longer life span, holding retirement fixed, raises the social security wealth of the aged despite a reduction in yearly benefits. Since the present value of tax revenues is fixed, this increase in social security wealth must be financed by a larger trust fund.

Kaplan [1976] calculated that for the United States the current social security deficit was about 1.4 trillion dollars.[9] In 1976 NNP equaled $1,511.8 trillion. Assuming a capital-output ratio of three gives us a more direct estimate of the partial equilibrium effect, simply the deficit divided by the capital stock. This figure equals 30.87 percent, somewhat smaller than those presented in Table I. (Assuming a 2.5 capital-output ratio yields 37.04 percent.)

The appeal of this partial equilibrium calculation lies in the fact that it does not require specifying tastes. However, as will be made clear below, the general equilibrium effects may be substantially smaller than the partial equilibrium impact. The computational findings for our particular utility functions caution against relying too heavily on partial equilibrium results.[10]

IV. General Equilibrium Changes in Capital Intensity

In general equilibrium, changes in the wage, interest rate, and age of retirement feed back to affect earnings, benefits, and consumption profiles. Ignoring issues of retirement and social security yield for the moment, we see that changes in factor returns will have both income and substitution effects on consumption. On the one hand, the lower net wage and higher net rate of interest reduce lifetime wealth and presumably savings when young; on the other hand, the higher rate of interest reduces the price of old age consumption and induces more savings by the young. In a life cycle model where the young substantially outnumber the old, their savings behavior is, of course, crucial to the final outcome. It appears, then, that the general

equilibrium changes can be either larger or smaller than those in partial equilibrium.

Turning to the general equilibrium calculation, we differentiate (12) totally with respect to the security tax rate θ (see Appendix A). Again we evaluate the percentage response of the capital stock to a 10 percent social security tax starting from an initial zero social security equilibrium. The final formula for the percentage change in the capital stock assumes a constant returns to scale production function; the actual calculations assume a Cobb-Douglas technology. Equation (12) gives the capital stock supplied by life cyclers as a function of factor returns, growth and taste parameters, and retirement and death ages. Given factor returns, growth parameters, and the age of retirement, there is a corresponding demand by firms for capital based on their production function. In equilibrium, the supply of capital by households equals the demand for capital by firms at the prevailing factor returns. To insure an initial equilibrium, we select ρ, the time preference parameter, such that the capital stock supplied by life cyclers facing the corresponding net interest rate (the net wage is normalized to one), retirement age, and other parameters equals the capital stock demanded by firms assuming the Cobb-Douglas technology. Specifically, we choose a value for the net interest rate and a set of values for all of the parameters of the model except ρ. We then calculate the demand for capital by firms and adjust the supply of capital by households to that demand by altering the value of ρ.

In the differentiation, R, the retirement age, is taken to be a function of W_N, defined as the net wage W, less social security taxes and benefits, i.e., the wage facing the worker for post-age R labor effort; $W_N = W(1-\theta) - \beta$. A retirement elasticity with respect to W_N is developed in the final formula:

$$(14) \qquad \frac{dK}{K} = \left(\frac{Q\gamma(ESF_1 + RF_2 + F_3LSCr) + H_2G}{(g-r)\,KG/W + \Psi(F_1 + RQF_2) + F_3\lambda r} \right) d\theta.$$

Despite the large number of terms, (14) permits some insight into the general equilibrium response. First, note that Q is the retirement elasticity and Ψ and λ are, respectively, the elasticities of the wage and interest rate with respect to capital. When all these terms are zero; i.e., when retirement and factor rewards are invariant to changes in the capital stock, (14) is identical to the partial equilibrium effect described above. E and C are, respectively, elasticities of the wage and interest rate with respect to the labor force. From the bracketed term in the numerator of (14), it is clear that earlier retirement affects the capital stock, in part, by raising the wage rate and lowering the interest

TABLE II

EFFECTS OF SOCIAL SECURITY ON THE STEADY STATE CAPITAL STOCK

Net rate of interest	Age of retirement	Rate of time preference	Retirement effect	Replacement effect	Total effect
0.04	42	0.0070	0.043	−0.260	−0.210
	45	0.0028	0.063	−0.258	−0.189
0.05	42	0.0160	0.047	−0.267	−0.213
	45	0.0120	0.066	−0.263	−0.190
	50	0.0058	0.147	−0.258	−0.104
0.06	42	0.0232	0.048	−0.267	−0.212
	45	0.0196	0.068	−0.263	−0.189
	50	0.0138	0.148	−0.256	−0.102

Note:
Elasticity of marginal utility: 1
Age of death: 55
Elasticity of retirement: 0.1
Interest income tax rate: 0.5
Wage income tax rate: 0.3
Social security tax rate: 0.1
$r = n + g$
$n = g$.

rate. Examination of the denominator of (14) for fixed retirement (Q equal to zero) shows that general equilibrium changes in the wage will magnify rather than dampen the partial equilibrium capital stock reduction. The term ΨF_1 captures this magnification; its sign is opposite to that of the other terms in the denominator.

In the calculations presented below we consider, in turn, replacement, retirement, and yield effects as well as variations in tastes. The calculations assume a life span of 55 years and equal rates of population and productivity growth. A retirement elasticity of 0.1 is assumed throughout. The rationale for this figure is the following: A two- to three-year reduction in a forty-five-year working period in response to a 50 percent reduction in the post-retirement wage (assuming a benefit-wage replacement ratio of 40 percent and a 10 percent social security tax) yields elasticities ranging from 0.089 to 0.133. Hence, 0.1 appears to be a historically plausible figure. The reader is of course free to apply his own subjective retirement elasticity to the formula.

Table II reports replacement and retirement effects for economies initially at an $r = n + g$ steady state. In this calculation the elasticity of marginal utility l equals 1, the logarithmic case. Other values for l will be explored below. The values of the rate of time

TABLE III

STOCK OF CAPITAL

Rate of interest	Age of retirement	Rate of time preference	Capital stock of pre-retirees	Capital stock of retirees	Total stock of capital
0.04	42	0.0070	180.35	36.90	217.25
	45	0.0028	201.05	25.18	226.23
0.05	42	0.0160	132.58	26.74	159.32
	45	0.0120	147.52	18.05	165.57
	50	0.0058	169.05	5.56	174.61
0.06	42	0.0232	101.73	20.10	121.83
	45	0.0196	111.79	13.34	125.13
	50	0.0138	127.37	4.02	131.59

Note:
$n + g = r$
$n = g$
Age of death: 55
Interest income tax: 0.5
Wage income tax: 0.3
Elasticity of marginal utility: 1.

preference that equate demands and supplies for capital in the initial equilibrium are also indicated. Changes in the capital stock are presented only for those economies that satisfy the initial equilibrium condition for a positive time preference parameter. For calculating the pure replacement effect the elasticity of retirement Q in (14) is set to zero; similarly in measuring the retirement effect by itself, H_2 is set to zero. Table III gives the initial level of capital as well as its distribution among the working and retired population for different parameter values. Table IV presents savings profiles for a life cycler prior to and after the introduction of social security for selected parameters values. Throughout the tables the initial age zero net wage is normalized to one.

The results of Table II suggest a general equilibrium reduction in the steady state capital stock ranging from 10 to 21 percent. The retirement effects can be quite important, offsetting by over half the replacement effects of social security for certain parameter values. The theoretical conjecture that the retirement effect may be negative is not confirmed in Table II; all retirement effects are positive. Assuming no retirement response at all, we see that the replacement effect is at most 27 percent; 27 percent, though smaller than Feldstein's 38 percent figure, is still quite a large change. The figures for

TABLE IV

PATTERNS OF LIFE CYCLE SAVINGS

Age	No social security	Social security in partial equilibrium	Social security in general equilibrium
0	0.329	0.229	0.304
5	0.414	0.263	0.374
10	0.511	0.302	0.444
15	0.618	0.335	0.502
20	0.732	0.353	0.532
25	0.849	0.337	0.511
30	0.961	0.276	0.400
35	1.056	0.145	0.141
40	1.117	−0.089	−0.347
45−	1.118	−0.471	−1.187
45+	−1.962	−0.347	−0.613
50	−3.398	−0.886	−1.765
55	−5.424	−1.705	−3.621

Note:
Rate of interest: 0.04
Age of retirement: 45
Rate of time preference: 0.012
$r = n + g$
$n = g$
45− and 45+ correspond to instants prior to and after age of retirement.
Elasticity of marginal utility: 1.

a forty-five-year working period are quite similar, independent of the initial rates of interest and time preference. If we take these numbers as the most reliable, the steady state reduction in the capital stock is about 19 percent with the retirement effect offsetting the replacement effect by about 7 percent.

In contrast to the partial equilibrium findings of Table I, Table II indicates that the reduction in the steady state capital stock is smaller the larger the initial age of retirement. However, different initial interest rates and the associated time preference parameters appear to have little effect on the results. Table III reveals that the initial levels of the capital stock are much more sensitive to starting parameter values than are the percentage change calculations of Table II.

So far we have reported only results that assume a social security tax yield equal to the net rate of interest. Table V considers non-(private) golden rule assumptions. The yield effect is computed as the change in the replacement effect when the term $-H_3H_5$ is included in forming H_2; when $r = n + g$, $H_3 \cdot H_5$ equals zero.

TABLE V

YIELD EFFECT OF SOCIAL SECURITY ON THE STEADY STATE CAPITAL STOCK

Growth rate of population plus productivity $(n + g)$	Rate of time preference	Yield effect	Retirement effect	Replacement effect	Total effect
0.03	0.0234	0.106	0.043	−0.312	−0.158
0.04	0.0178	0.053	0.054	−0.286	−0.172
0.05	0.0120	0.000	0.066	−0.263	−0.190
0.06	0.0064	−0.056	0.081	−0.246	−0.213

Note:
Net rate of interest: 0.05
Age of retirement: 45
Elasticity of marginal utility: 1
$n = g$
Social security tax: 0.10.

The numbers presented indicate that even a small (1 percent) differential in yields can have a substantial influence on the final steady state outcome. All yield effects have the theoretically indicated sign. Higher than interest yields on social security tax contributions lead to more consumption and reduce accumulation while lower yields promote savings. A 1 percent yield differential appears to have about a 5 percent impact on the final steady state capital stock. However, Table V also indicates that higher social security yields are associated with larger retirement and smaller replacement effects, leaving the general equilibrium changes still in the neighborhood of 19 percent.

Comparing the replacement effects of Tables II and V with the partial equilibrium changes in Table I, we see that in general equilibrium the response of savings to a higher interest rate plays a major role in dampening the partial equilibrium impact. In Table IV this savings response is depicted for a typical life cycler. The table presents savings patterns for three situations: zero social security, social security assuming only partial equilibrium changes in the capital stock, and social security allowing for general equilibrium changes in factor returns but no retirement or yield effects. The table indicates a strong response of savings to changes in factor returns for this particular set of preferences. For the utility function under consideration the reaction of savings to the interest rate is neatly parameterized by l, the elasticity of marginal utility. High elasticities of marginal utility flatten out consumption profiles and produce a smaller savings re-

TABLE VI

STEADY STATE CAPITAL REDUCTIONS UNDER VARIOUS PREFERENCE STRUCTURES

Elasticity of marginal utility	Rate of time preference	Retirement effect	Replacement effect	Total effect
0.50	0.0310	0.016	−0.154	−0.135
0.75	0.0216	0.043	−0.213	−0.165
1.00	0.0120	0.066	−0.263	−0.190
1.25	0.0026	0.086	−0.307	−0.212

Note:
Net rate of interest: 0.05
Age of retirement: 45
$r = n + g$
$n = g$.

sponse to higher interest rates. Low elasticities of marginal utility imply the opposite characteristics. Table VI presents capital stock changes that assume different elasticities of marginal utility. For an elasticity of 1.25 the replacement effect is 31 percent, while an elasticity of 0.5 entails only a 15 percent replacement impact in general equilibrium.

V. ALTERNATIVE SPECIFICATIONS AND SOME MACRO ISSUES

The analysis above has highlighted some aspects of social security's impact on savings to the exclusion of others. First, as mentioned above, capital market imperfections may imply a substantially different savings response by the young from that posited above. Second, social security may reduce the uncertainty of old age consumption and retirement savings by providing an indexed real annuity.[11] A third important issue raised by Barro is the extent to which the intergenerational transfers associated with unfunded social security led to offsetting private transfers with no reduction in aggregate savings.[12] The argument here is that transfers from the young to the old through social security simply replace private transfer from young to old. Alternatively, forced social security transfers from young to old may result in offsetting transfers from the old to the young in the form of bequests. Unfortunately, there is little empirical evidence to resolve this question. By ignoring private intergenerational transfers in our analysis, we do not mean to belittle this issue; rather, we have attempted here to provide an upper bound for social security's impact on the steady state capital stock.

At the macro level the notion that the U. S. experience under social security can be understood as a movement across steady states with a concomitant decline in the capital stock begs a number of issues. Chief among these is the historical ability of the economy to transform the consumption expenditures that were generated by the unfunded social insurance system into savings while maintaining full employment. In other words, in the absence of social security would the Federal deficit have stayed the same or risen to match the historic rise in the social security deficit? Much of the historic depletion of the social security trust fund was due to redistributional considerations. The early dissolution of the link between tax payments and benefits came about because of a desire to aid elderly citizens who had never paid into the system as well as dependent wives and widows who had never worked. Hence, it is appropriate to inquire whether this type of redistribution would not have taken place in any case with similar consequences for the historic level of aggregate savings.

Finally, even if agreement could be reached on the statement that social security has depleted U. S. wealth holdings, the openness of the U. S. economy argues against assigning the entire depletion to the capital stock in place in the United States.

VI. Summary and Conclusion

These caveats are important for viewing our results in proper perspective. We have provided evidence above that supports the proposition that social security *could* have caused a *major* reduction in the capital stock. The life cycle model with its retirement effects suggests a 20 percent steady state reduction in general equilibrium, although a substantially greater partial equilibrium reduction. While we caution that our results may be sensitive to the particular preference structure considered, it appears that reasonable retirement responses can only partially offset the replacement effect of social security on the capital stock. In addition, the analysis has suggested how general equilibrium changes in factor returns can magnify or dampen the partial equilibrium effects presented above.

To conclude, we feel that the final resolution of social security's effect on capital accumulation will require additional empirical work at both the micro and macro levels. The results for our simplified life cycle economy indicate that the impact of unfunded social security on capital intensity can be quite large and, hence, deserve to be taken seriously.

APPENDIX A

Solution of differential equations (5) and (6) in the text are given by

$$K_L(t) = W(1-\theta)\left(\frac{e^{rt}-e^{gt}}{r-g}\right) + vI(e^{(r-\rho/l)t} - e^{rt}) \tag{A.1}$$

and

$$K_M(t) = \frac{-B(e^{gt} - e^{gL+r(t-L)})}{r-g} - vI(e^{xD+rt} - e^{(r-\rho)/lt}), \tag{A.2}$$

where

$$x = [r(1-l) - \rho]/l$$

and

$$v = \frac{1}{1-e^{xD}}.$$

Integrating $K_L(t)$ and $K_M(t)$ over age cohorts yields (A.3) (expression (7) in the text):

$$K = \frac{W}{r-g}[H_1 - \theta H_2 + H_3 H_4], \tag{A.3}$$

where

$$H_1 = \left(\frac{1-e^{(r-u)R}}{u-r}\right) - \left(\frac{1-e^{-nR}}{n}\right),$$

$$H_2 = \frac{L}{M}\left[\left(\frac{e^{-nR}-e^{-nD}}{n}\right) - \left(\frac{e^{(r-u)R}-e^{(r-u)D}}{u-r}\right)e^{(g-r)D}\right] + H_1 - H_3 \cdot H_5,$$

$$H_3 = \left[\frac{e^{zD}-1}{z} - \left(\frac{1-e^{(r-u)R}}{u-r}\right) - \left(\frac{e^{(r-u)R}-e^{(r-u)D}}{u-r}\right)e^{xD}\right],$$

$$H_4 = \frac{1-e^{(g-r)R}}{1-e^{xD}},$$

$$H_5 = -H_4 + \frac{L}{M}\left[\frac{e^{(g-r)R}-e^{(g-r)D}}{1-e^{xD}}\right],$$

$$u = n + g,$$

$$z = [(r-u)l - \rho]l$$

Differentiating (A.3) with respect to θ yields

$$\frac{dr}{d\theta}K + (r-g)\frac{dK}{d\theta} = \frac{dW}{d\theta}F_1 + W\frac{dH_2}{d\theta} - WH_2 + WH_3\frac{dH_4}{d\theta} + H_4\frac{dH_3}{d\theta}, \tag{A.4}$$

where

(A.5) $F_1 = H_1 + H_4H_3$ (see text for definition of symbols)

(A.6) $$\frac{dH_1}{d\theta} = (e^{(r-u)R} - e^{-nR})\frac{dR}{d\theta} + \left[\frac{(r-u)(R-1)e^{(r-u)R} + 1}{(r-u)^2}\right]\frac{dr}{d\theta},$$

(A.7) $$\frac{dH_3}{d\theta} = (e^{xD} - 1)\frac{dR}{d\theta}e^{(r-u)R} + J\frac{dr}{d\theta},$$

(A.8) $$\frac{dH_4}{d\theta} = (r-g)ve^{(g-r)R}\frac{dR}{d\theta} + \left[Re^{(g-r)}v + v^2\frac{(1-l)D}{l}(1 - e^{(g-r)R})e^{xD}\right]\frac{dr}{d\theta}.$$

Let $f(L,K)$ stand for the production function:

(A.9) $$W_g = f_L(L,K)$$

(A.10) $$\frac{dW_g}{d\theta} = f_{LL}\frac{dL}{dR}\frac{dR}{dWn}\frac{dWn}{d\theta} + f_{LK}\frac{dK}{d\theta}.$$

Taking R as a function of $Wn = W(1-\theta) - B$ and setting $\gamma = (M+L)/M$ implies that

(A.11) $$\frac{dWn}{d\theta} = \frac{dW}{d\theta} - W\gamma.$$

(A.10) and (A.11) give

(A.12) $$\frac{dW}{d\theta} = \frac{-W\gamma EQS}{1-EQS} + \frac{(1-tw)f_{LK}}{1-EQS}\frac{dK}{d\theta},$$

where

$$S = \frac{dL}{dR}\frac{R}{L},$$

$$E = \frac{f_{LL}L}{W},$$

$$Q = \frac{dR}{dWn}\frac{Wn}{R}.$$

Similarly we can find that

(A.13) $$\frac{dr}{d\theta} = \frac{-rCQS\gamma}{1-EQS} + H_6\frac{dK}{d\theta},$$

where

$$H_6 = \frac{rCQS}{W}\frac{(1-tw)f_{LK}}{(1-EQS)} + (1-tr)f_{KK},$$

$$C = \frac{f_{LK}L}{r}.$$

Substituting the above expressions into (A.4), using the additional terms defined below, and dividing by K we obtain (a) in the text. Additional terms entering equation (19) are

$$\Psi = f_{KL}L/W$$

$$\lambda = F_{LK}L/r,$$

$$F_2 = \frac{H_3(r-g)e^{-(r-g)R}}{1-e^{xD}},$$

$$F_3 = \frac{-K}{W} + \frac{R}{r-g}F_2 + H_4J + F_4 + \frac{H_4H_3D}{e^{-xD}-1}\frac{(1-l)}{l}$$

$$F_4 = \frac{[(r-u)R-1]e^{(r-u)R}+1}{(r-u)^2},$$

$$G = 1 - EQS$$

$$J = \frac{[(zD-1)e^{zD}+1]}{lz^2} - F_4 + \frac{[(R(r-u)-1)e^{(r-u)R} - (D(r-u)-1)e^{(r-u)D}]}{(r-u)^2}e^{xD} + \left[\frac{e^{(r-u)R}-e^{(r-u)D}}{(r-u)}\right]e^{xD}\frac{(1-l)D}{l}.$$

APPENDIX B

The deficit (DEF) of an unfunded social security system is defined as the present value of future benefits less future taxes for the living population:

$$\text{(B.1)}\quad DEF = \int_0^D PVB(a)e^{-na}da - \int_0^R PVT(a)e^{-na}da,$$

where $PVB(a)$ and $PVT(a)$ are the present value of benefits and taxes for an individual age a.

Prior to retirement

(B.2)

$$PVB(a) = \int_R^D Be^{(g-r)(t-a)}dt = B\left(\frac{e^{(g-r)R}-e^{(g-r)D}}{r-g}\right)e^{(r-g)a},$$

and after retirement

(B.3)

$$PVB(a) = \int_a^D Be^{(g-r)(t-a)} = B\left(\frac{e^{(g-r)a} - e^{(g-r)D}}{r-g}\right)e^{(r-g)a}.$$

$PVT(a)$ is given by

(B.4)

$$PVT(a) = \int_a^R We^{(g-r)(t-a)} = \theta W\left(\frac{e^{(g-r)a} - e^{(g-r)R}}{r-g}\right)e^{(r-g)a}.$$

Using (B.1)–(B.4), setting $r = n + g$, and recalling that

(B.5)
$$B = \frac{1 - e^{-nR}}{e^{-nR} - e^{-nD}} = \frac{L}{M},$$

we obtain

(B.6)
$$DEF = \frac{\theta W}{n}\left(R + (R - D)\frac{L}{M}e^{-nD}\right),$$

the partial equilibrium effect discussed in the text.

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NOTES

1. For a comprehensive review of the growing literature on social security and retirement see Cambell and Cambell [1976] and Pellechio [1978].
2. Feldstein [1976] is the pioneering work in this area and contains a survey of the literature. We consider here the pure life cycle model without intergenerational transfers in order to indicate the maximum possible reduction in the capital stock from unfunded social security.
3. Assuming consumption at every age is a normal good.
4. Kotlikoff [forthcoming], presents a micro econometric study of the replacement, yield, and retirement effects.
5. See, for example, Tobin [1967].
6. See Appendix A.
7. This is a private golden rule condition, not a social golden rule, since r is the net, not the gross, rate of return. The current U. S. situation appears to approximate $r = n + g$.
8. See Tobin [1967], p. 255.
9. See Kaplan [1976].
10. Indeed, once general equilibrium changes occur, a partial equilibrium computation (such as that just performed for the United States), which uses current values of r, R, L, and M, may be substantially biased away from the true partial equilibrium effect.
11. See Diamond [1976].
12. See Barro [1974].

REFERENCES

Barro, Robert J., "Are Government Bonds Net Wealth?" *Journal of Political Economy*, LXXXII, No. 6 (Nov./Dec. 1974), 1095–119.

Cambell, Colin D., and Rosemary G. Cambell, "Conflicting Views on the Effect of Old-Age and Survivors Insurance on Retirement," *Economic Inquiry*, XIV (Sept. 1976) 369–88.

Diamond, Peter A., "A Framework for Social Security Analysis," paper presented to NSF/NBER Conference on Taxation at Stanford, January 1976.
Feldstein, Martin, "Social Security, Induced Retirement, and Aggregate Capital Accumulation," *Journal of Political Economy,* LXXXII, No. 5 (1974), 905–26.
——, "The Optimal Financing of Social Security," Harvard Institute of Economic Research Paper No. 388, November 1974.
——, "Social Security and Saving: The Extended Life Cycle Theory," *American Economic Review,* LXVI, No. 2 (May 1976), 77–87.
Kaplan, Robert S., "Financial Crisis in the Social Security System," Domestic Affairs Study No. 47, June 1976, American Enterprise Institute for Public Policy Research, Washington, D.C.
Kotlikoff, Laurence J., "Testing the Theory of Social Security and Life Cycle Accumulation," *American Economic Review,* forthcoming.
Pellechio, Anthony, "Social Security and Retirement Behavior," Ph.D. thesis, Harvard University, 1978.
Tobin, James, "Life Cycle Savings and Balanced Growth," in *Ten Economic Studies in the Tradition of Irving Fisher,* William J. Fellner, ed. (New York: J. Wiley, 1967).