

Generational policy and aging in closed and open dynamic general equilibrium models

by

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Abstract

This chapter examines the micro- and macroeconomic effects of generational policies using closed and open general equilibrium dynamic life-cycle models. The models illustrate the broad array of demographic, economic, and policy issues that can be simultaneously incorporated within today's computable models of economic growth. The list includes country-specific tax, spending, social security, healthcare policy, and deficit policy, age- and country-specific mortality, age-specific fertility, age-specific morbidity, lifespan uncertainty, age- and skill-specific emigration and immigration, earnings inequality driven by skill differences and idiosyncratic labor earnings uncertainty, capital adjustment costs, international trade, international capital flows, trade specialization, and trade policy.

The chapter begins with the benchmark dynamic OLG simulation model of Auerbach and Kotlikoff (1987), discusses various advances in OLG simulation modeling, and then presents two applications. The first is a closed economy model, calibrated for Germany, that features idiosyncratic labor earnings uncertainty and changes in demographics. After running the model through a number of policy simulations, we turn to an open economy model, featuring five regions (the U.S., Europe, Japan and other Asian countries, China, and India) producing six goods, some of which are traded. We use this model to quantify how economies will transition

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over time, how wage inequality will evolve, how tax rates will change in light of societal aging, and how various unilateral and multilateral policy reforms impact the six regions.

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1 Introduction

This paper examines the micro- and macroeconomic effects of generational policies using closed and open general equilibrium dynamic life-cycle models. The models illustrate the broad array of demographic, economic, and policy issues and factors that can be simultaneously incorporated within today's computable models of economic growth. The list includes country-specific tax, spending, social security, healthcare policy, and deficit policy, age-, cohort-, and country-specific specific mortality, age-specific fertility, age-specific morbidity, lifespan uncertainty, age- and skill-specific emigration and immigration, earnings inequality driven by skill differences and idiosyncratic labor earnings uncertainty, capital adjustment costs, international trade, international capital flows, and trade specialization.

Including these first-order elements, either in full or in part, into a single model is a significant programming challenge. But there is no substitute for such endeavors if one is to achieve a proper sense of the magnitude and timing of policy effects as well as their impacts on the welfare of different generation and the efficiency of the economy. The need to solve for equilibrium paths of what are very high (roughly 250) order difference equations models that contain upwards of one million variables raises the question of whether the models work, whether they produce a unique equilibrium for a given calibration and policy specification, and whether they are tractable.

The answer is that these "black boxes" do work insofar as one can verify that all equations are satisfied, to very high degrees of precision, that they appear to be unique insofar as deviations in the initial guesses of variables used in the numerical solution algorithms do not alter final solutions, and that they are tractable insofar as one can readily understand the models' findings and trace them back to the policies being run and the microeconomic behavior underlying the models' policy responses.

The Gauss-Seidel iterative solution technique used to solve for the equilibrium transition paths was introduced by Auerbach and Kotlikoff (1983). Prior to their work, Tobin (1967), Kotlikoff (1979), and others had used iteration to solve for steady states of complex life-cycle economies and Summers (1981) had assumed myopic, i.e., irrational, expectations, in approx-

imating transition paths while avoiding the computational challenge associated with finding a rational expectations (perfect foresight, in this context) solution. A key advantage in assuming rational agents is that they consider fully what's coming, be that changes in factor prices or changes in policies, both of which alter their incentives to work and save in the present as well as the future. And with overlapping, finite-lived generations, the economy always has young agents who are looking beyond the horizons of their elders. This forward looking behavior connects those now alive to those alive in the infinite future and makes reactions to current policy dependent on future policy as well as on the reactions of future agents to future policy. This feature of dynamic transition paths teaches us that one can't examine the short run independent of the long run or cogently discuss short-run policy, while avoiding long-run policy. This becomes immediately apparent when one visualizes all future markets as clearing immediately as in Arrow's contingent claims model. Leaving out future markets or policy would be like ignoring the market for, and taxation of, apples in a static model in which firms supply and agents demand apples, bananas, and peaches.

The two overlapping generations models used here to illustrate the power of dynamic life-cycle simulation to answer critical economic questions are descendants of Auerbach and Kotlikoff (1987).¹ As we'll discuss, various extensions of the original Auerbach-Kotlikoff model have been introduced in recent years.

The following section introduces the general structure and discusses the modeling of population dynamics in a national and international context, endogenous retirement choice, idiosyncratic income uncertainty and intragenerational risk sharing. Then we present a closed economy model for Germany and simulate the economic consequences of various pension reforms in a stochastic set up. Section four introduces a deterministic multi-country model of the world economy. The focus here is on international spillover effects of pension reforms and the impact on the world capital market.

¹Alternatively, quantitative studies such as Jaag, Keuschnigg and Keuschnigg (2010) or Hejdra and Romp (2010) are in the tradition of Blanchard (1985). While this approach allows one to study the effects of demographic changes analytically in general equilibrium, the implicit restrictions limit its usefulness for quantitative applications.

2 Preliminaries: Modeling of aging, retirement and idiosyncratic income risk

The Auerbach and Kotlikoff (1987) model (henceforth the AK model) builds on the overlapping generations structure of Samuelson (1958) and Diamond (1965), but extends it along several dimensions to obtain more realistic quantitative assessments. First and foremost, the model accommodates an arbitrary number of overlapping generations. But incorporating more generations than two, and even just two under some circumstances, produces a non-linear difference equation that must be solved numerically. Auerbach and Kotlikoff's solution method entailed a) assuming the economy reached a steady state by a specified future date and then b) guessing the economy's time path of factor prices and fiscal variables, determining household supplies of productive factors, treating the aggregate values of these supplies as aggregate factor demands (since demands equal supplies in equilibrium) and using these aggregate factor demands to generate a new path/guess of factor prices and fiscal variables. This iterative Gauss-Seidel technique of using the prior guessed path of the economy to derive a new guessed path of the economy is repeated as often as needed to reach convergence of the economy's path to many decimal places of accuracy. In solving for household factor supplies (supplies of capital and labor), Auerbach and Kotlikoff used numerical iteration as well. Thus, their method entails inter-loop (micro-economic) as well as outer-loop (macro-economic) convergency.

The original AK model distinguishes between 55 overlapping generations (i.e. ages 21 to 75). Maximizing an intratemporal CES and an isoelastic intertemporal utility function, households decide in each period about consumption, labor supply and saving taking into account an intertemporal budget constraint, which ensures that the present value of lifetime resources (i.e. financial plus full time human wealth gross of the present value of lifetime government transfers and net of present value of lifetime taxes) does not exceed the present value of expenditures on consumption and leisure. Government policy enters the household budget constraint through the present values of transfers and taxes as well as through the household's first order conditions.

The original AK model, developed in 1980, presented at an NBER tax-policy simulation conference in 1981, and published in 1983, incorporated no uncertainty. Households work until

passing an exogenously specified "retirement age". Afterwards they receive social security benefits until they die. Agents face no liquidity constraints, so that they might accumulate debt during young age which they pay back later in life. The model is able to replicate the social security system in detail by the specification of individual contribution payments and pension benefits. Since at the end of each period the oldest cohort alive dies, people know exactly their remaining life span. Due to the absence of a specific bequest motive there are neither intended nor unintended bequest in the original model. Since the remaining lifetime is varying across cohorts living in a specific year, fiscal reforms have a different impact on the budget constraints of old, young, and future cohorts. Typically, the initial long run equilibrium is calibrated to represent the existing fiscal system. After a policy reform is announced or enacted, the model computes a transition path to a new long run equilibrium.

Given the model's solution, researchers are able to evaluate the transitional growth effects for the macroeconomy as well as the distributional consequences of the considered reform for different current and future generations. In addition, it is also possible to quantify the aggregate efficiency consequences of a specific reform. For this purpose, Auerbach and Kotlikoff (1987) introduce the so called "Lump sum redistribution authority" (LSRA) which compensates existing generations after the reform with lump-sum transfers and taxes, so that they end up at their pre-reform welfare. Due to the absence of uncertainty, bequest and liquidity constraints, individuals can perfectly provide for their old-age consumption by means of private saving. Consequently, a mandatory individualized funded pension program that doesn't redistribute across generations would only replace private savings with saving in the public pension fund but would have no real effects in the economy. On the other hand, a paygo program that redistributes toward the initial elderly from their children and future descendants crowds out the capital stock and redistributes resources from young and future generations towards the elderly. Consequently, a move from a paygo to a funded program that entails intergenerational redistribution will always increase the capital stock of the economy and redistribute resources from current towards future generations.

Various studies that have been carried out with the original model during the 1990's are discussed in Kotlikoff (2000). The focus of this chapter, however, is on three more recent innova-

tions in the Auerbach-Kotlikoff model, namely aging in an international context, endogenous retirement, and idiosyncratic uncertainty. In the remaining section we will introduce these issues separately.

Auerbach and Kotlikoff (1987) as well as Auerbach et al. (1989) already present calculations that quantify the impact of population aging for the government and the macroeconomy in the US, Japan, Sweden and Germany. These studies clearly pointed out the dramatic increase in social security contributions and tax rates as well as a rise in wages and fall in interest rates due to aging. However, within the structure of the original single country model, population aging could only be captured by an exogenously specified population vector which changes from year to year. Consequently, aging did not change the individual consumption, saving and bequest behavior, nor did the analysis include international repercussion effects. In the following years various studies have included age-specific survival probabilities and an uncertain life span, see Broer and Lassila (1997). However, since these studies assume perfect annuity markets, the remaining assets of those who have died are distributed to the surviving members of the respective cohort. Consequently, the economic effects of these models did not really change compared to the model with certain life span. The framework of Kotlikoff et al. (2007) features a much more detailed mapping of the demographic process in the U.S. During their child-bearing years, agents give birth each year to fractions of children. This means of finessing marriage and family formation permits the incorporation of changes through time in age-specific fertility rates and the calibration of the model's age-specific population shares to the official population forecasts. In addition, they also assume that agents care about their children's utility when the latter are young and, as a consequence, make consumption expenditures on behalf of their children. Therefore, the model delivers the hump in the consumption profile that appears during child-rearing years in the actual data. As in De Nardi, İmrohoroğlu and Sargent (1999), they include utility from leaving bequest and realistic mortality probabilities for agents. However, agents fail to annuitize their assets in old age. Consequently, agents gradually reduce their consumption due to the uninsurable lifespan uncertainty and leave desired and undesired bequests to their children when they die. While agents die at different ages and have children of different ages, their heirs also inherit at different ages. Agents who were born when

their parents were young receive inheritances later in their life than do their younger siblings.

The multi-region models of Fehr, Jokisch and Kotlikoff (2004, 2005, 2007, 2008) build directly on Kotlikoff et al. (2007). Besides the explicit provision for immigration, the demographic transition at the national level and the household decision problem is modeled in a very similar fashion. However, the aging processes of U.S., Europe, and various Asian countries are now interlinked via the international capital market. Each simulation starts from initial conditions that include the population structure of the respective countries in year 2000. Due to the diverging population dynamics and fiscal systems, the growth paths as well as the asset prices of the three considered regions differ. Since the model includes a detailed fiscal system, aging increases overall taxes, so that capital is crowded out and the world interest rate rises. The latter result stands in stark contrast to some other recent multi-country OLG models such as Brooks (2003), Saarenheimo (2005) or Börsch-Supan et al. (2006), which predict that aging will reduce the rate of return on the international capital market during the next 50 years since labor supply will become more scarce relative to capital. Fehr, Jokisch and Kotlikoff (2004, 2005, 2007, 2008) also account for intracohort heterogeneity. In addition to three different skill levels, they also distinguish between native and foreign individuals. Immigrants are also split into these income classes permitting them to simulate the arrival of immigrants with different stocks of human and physical capital. Since especially high-skilled immigrants are known to be net tax payers to the public system, selective immigration is often offered as a solution to the demographic transition under way in the industrialized world, see Storesletten (2000). However, the simulations in Fehr, Jokisch and Kotlikoff (2005) show that even a significant expansion of immigration, whether across all skill groups or among particular skill groups, will have only a minor impact on the major capital shortage and tax hikes that can be expected along the demographic transition. While the prospects with respect to immigration seem to be quite frustrating, the model's predictions are dramatically altered when China is added to the picture. Even though China is aging rapidly, its saving behavior, growth rate, and fiscal policies are currently very different from those of developed countries. As Fehr, Jokisch and Kotlikoff (2007) demonstrate, China might eventually become the developed world's savior with respect to its long-run supply of capital and long-run general equilibrium prospects, if successive cohorts of Chinese continue

to save like current cohorts, if the Chinese government can restrain growth in expenditures, and if Chinese technology, education and consumption levels ultimately catch up with those of the West and Japan. China's economic development and savings do not only reduce world interest rates but also increase significantly wages in industrialized countries. Finally, Fehr, Jokisch and Kotlikoff (2008b) also include India, distinguish different production sectors and model imperfect substitution between skill-specific labor inputs. Their study clearly indicates that aging and globalization increase the skill premium for high-skilled workers due to the rising relative world supply of low-skilled labor.

Increases in retirement ages are often seen as an alternative to tax increases or benefit cuts. Consequently, among others, Fehr (2000), Sánchez-Martín (2010) as well as Kotlikoff et al. (2007) also quantify the macroeconomic and distributional consequences of an increase in the eligibility age for social security. However, retirement choice in these models is very artificial. Given an exogenous age when they start receiving pension benefits, agents can only decide at what age they quit working. In order to achieve retirement exactly at the eligibility age for social security, either a significant drop in productivity or a dramatic increase in marginal labor income taxes is assumed at the eligibility age. This approach has mainly two disadvantages. First, the drop in individual productivity around retirement is at odds with empirical evidence, which indicates only a modest decline in productivity between ages 60 and 70, see the estimates in Altig et al. (2001) or French (2005). Second, and even more important, since agents have no choice when to claim social security, social security rules which affect early retirement can't be captured by these models.

Consequently, recent studies have introduced models where individuals have a labor-leisure choice in each working year, but also optimize the retirement age when they quite working and start to receive their pensions. Technically, the household optimization problem is solved in two stages. Given a price vector from the supply side of the economy, individuals first compute their optimal consumption and leisure path for alternative retirement ages. Then the retirement age which yields the highest utility level is selected on the second stage. Due to the evaluation of various alternatives and the discrete jumps in the aggregate variables, the computation is quite complicated. Fehr, Sterkeby and Thøgersen (2003) analyze early retirement incentives

of the Norwegian pension system in a model that distinguishes five income classes within a generation. The simulations indicate that reforms which increase the retirement age also have a positive long run welfare impact. Eisensee (2005) jointly determines fiscal sustainability and the retirement age in the US. The model distinguishes within a cohort between low, medium and high skilled labor and extends Fehr, Sterkeby and Thøgersen (2003) by including population aging and the transition path. However, Eisensee (2005) does not compare the welfare consequences of alternative policy options with endogenous retirement. The latter is done in recent studies by Diaz-Gimenez and Diaz-Saavedra (2009) and Sánchez-Martín (2010) as well as Fehr, Kallweit and Kindermann (2011) that explore various reforms aimed at improving the sustainability of the Spanish and German pension systems. These studies indicate that raising the normal retirement age (NRA) is an important policy instrument in an aging society. For example, in the German model by Fehr et al. (2011) a two-year increase in the NRA raises the long-run effective retirement age by one year, which induces in turn a decline in the contribution rate by 1.5 percentage points.

During the last decade, a new direction of quantitative research has extended the traditional AK-model by considering various sources of idiosyncratic economic risk, see Krueger (2006). Hubbard and Judd (1987) extend the AK model by including life span uncertainty and liquidity constraints. Since private annuity markets are missing, social security provides an insurance against longevity risk but at the same time increases welfare losses because it exacerbates the problems associated with borrowing constraints. However, Hubbard and Judd (1987) abstract from labor income uncertainty and precautionary savings are neglected, so that liquidity constraints are relevant for all young individuals. But a meaningful analysis of the insurance and liquidity effects of social security has to include income risk, so that various overlapping generations general equilibrium models have been developed in this direction. Imrohoroğlu et al. (1995) were the first to examine the optimality and welfare effects of alternative social security arrangements in a framework with stochastic employment opportunities. Agents supply labor inelastically when they are given the opportunity to work and otherwise receive unemployment benefits. After a mandatory retirement age, individuals rely on flat-rate pension benefits. In this framework, the welfare consequences of social security reflect the trade-off between the

(positive) insurance provision against income and longevity risk and the (negative) effects of stronger binding liquidity constraints. While the institutional setup is very favorable for social security, the calibrated initial equilibrium (without social security) lacks dynamic efficiency. When the growth rate of the economy exceeds the interest rate, it is not surprising that the introduction of social security increases the resources of all generations. In their follow up study, İmrohoroğlu et al. (1999) eliminate dynamic inefficiency by incorporating land as a fixed factor of production. In an economy with land the rise of the capital stock towards the golden rule level reduces the interest rate but also increases the price of land. The latter absorbs the savings of younger cohorts and confers capital gains on the owners of land, inducing higher consumption and ruling out the overaccumulation of capital. In this setting, the introduction of social security has again positive insurance and negative liquidity effects, but it also redistributes income across generations. The simulations indicate that an economy without social security provides the highest welfare for individuals. However, this result might only reflect the negative income effects of social security for future generations. Of course, the same critique also applies to other studies that analyze only the long-run consequences of gradual social security reforms such as Huggett and Ventura (1999) or Storesletten et al. (1999).

In order to provide a complete assessment of social security, one has to compute the transition path between steady states and separate intergenerational distribution from efficiency effects. A first study that adopts this approach is Huang et al. (1997) who compare two experiments where the existing unfunded social security system is eliminated and a private or a mandatory state-run funded system is introduced with all existing and transitional generations compensated. While both experiments yield a significant aggregate efficiency gain, the government-run funding scheme is preferred to privatization due to its superior insurance properties. De Nardi et al. (1999) extend this model by including realistic US demographics and variable labor supply. The latter allows them to analyze reforms where the tax-benefit linkage of the pension system is improved, which increases welfare in their framework. Conesa and Krueger (1999) simulate an immediate, a gradual and an announced elimination of social security and compute the political support for the three proposals in the initial year. Although for all cases considered agents would prefer to be born into the final steady-state, no proposal receives an initial voting

majority in the closed economy case. The political support is declining when intra-cohort heterogeneity is increasing due to the rising insurance gains from flat pensions. While Conesa and Krueger (1999) can explain why pension reforms are delayed in democratic systems, their study does not include efficiency calculations.

However, since the resulting welfare changes are not aggregated across individuals and generations, the overall efficiency effect is not explicitly determined. The latter is done by Nishiyama and Smetters (2007) who simulate a stylized 50-percent privatization of the US social security system. Again, the considered reform reduces labor supply distortions but also the insurance provision of the social security system. In order to isolate overall efficiency effects, the authors follow Auerbach and Kotlikoff (1987) by introducing a Lump-Sum Redistribution Authority (LSRA) which compensates initial agents and distributes the accumulated assets (i.e. efficiency gains) or debt (i.e. efficiency losses) to newborn and future agents. They find efficiency gains from privatization which amount to \$ 18.100 (in 2001 growth adjusted dollars) per household, if wage shocks could be insured privately. Consequently, if income uncertainty is perfectly insured, the loss in annuity provision is overcompensated by reduced labor market distortions and increased liquidity of younger households. However, when the reform is simulated with idiosyncratic labor income uncertainty, the aggregate efficiency effect of partial privatization turn negative, so that households lose \$ 2.400 or more than 5 percent of median income in 2001. This clearly indicates that the (positive) insurance effects of the US social security system dominate the distortionary effects on labor supply. Fehr and Habermann (2008) as well as Fehr, Habermann and Kindermann (2008) reach a similar conclusion for the German social security system. In contrast to the US system, benefits in the German system are strongly linked to former contributions. This institutional feature minimizes labor supply distortions but at the same time also reduces the insurance provision against income shocks. Simulations show that a more progressive system would yield a significant aggregate efficiency again, if all initial generations are compensated by LSRA transfers. Eliminating social security in Germany reduces aggregate efficiency as in Nishiyama and Smetters (2007). Fehr, Habermann and Kindermann (2008) compare the efficiency consequences in economies with rational and

hyperbolic consumers in order to isolate the commitment effect of social security.²

This is the point of departure for the model which will be discussed in the following section. It extends the previous studies by Fehr and Habermann (2008) as well as Fehr, Habermann and Kindermann (2008) in various directions. First, it includes a detailed demographic transition featuring the latest population forecasts in Germany. Second, it does not only consider temporary shocks in productivity and wage income but also permanent shocks in health in order to account for disability risk. Consequently, the model's social security system comprises an old-age pension and a disability insurance. Finally, our model allows for endogenous labor supply at the intensive and the extensive margin. As will be shown below, we are able to represent the current retirement pattern in Germany.

In the following section we will explain in detail the structure of the closed economy simulation model with idiosyncratic shocks.

3 The closed economy model for Germany

3.1 General model structure

3.1.1 Demographics and intracohort heterogeneity

We consider an economy populated by overlapping generations of individuals with the (exogenous) skill level $s \in \mathcal{S} = \{1, \dots, S\}$. The skill level s determines the individual productivity e_j and affects individual mortality which will also depend on the date of birth (i.e. labor market entry). Consequently, individuals of age $j \in (1, \dots, J)$ may live up to a maximum possible lifespan of J periods, where individual lifespan uncertainty is measured by $\psi_{j,t}^s \leq 1$, the period-dependent conditional survival probability from age $j - 1$ to age j of skill type s . At the beginning of each period t , a new generation enters the model where the population growth rate n_t depends on the fertility pattern.

Our model is solved recursively. Consequently, an age- j agent faces the individual state vector

$$z_j = (s, a_j, ep_j, \eta_j, d_j, o_j) \tag{3-1}$$

²Imrohoroğlu et al. (2003) have introduced quasi-hyperbolic discounting in the framework. Their study already indicates that social security may always serve as a commitment device for myopic individuals who do not adequately save for their retirement.

where $a_j \in \mathcal{A} = [0, \infty]$ denotes assets held at the beginning of age j and $ep_j \in \mathcal{P} = [0, \infty]$ defines agent's accumulated earning points for public pension claims. Whereas the skill class s can be interpreted as a permanent shock, agents are also exposed to idiosyncratic productivity shocks $\eta_j \in \mathcal{E} = [-\infty, \infty]$ which affect labor productivity. In addition, they are exposed to the risk of becoming severely disabled and therefore unable to work. The variable $d_j \in \mathcal{D} = \{0, 1\}$ indicates whether the agent is disabled ($d_j = 1$) or not whereas $o_j \in \mathcal{R} = \{0, 1\}$ changes from 0 to 1 in the moment the agent chooses to retire and therefore to become a regular old-age pensioner. Consequently, in each period t , the age- j cohort is fragmented into subgroups, according to the initial distribution at age $j = 1$ as well as mortality, population growth and optimal household decisions. Let $X_t(z_j)$ be the corresponding cumulated measure, so that

$$\int_{\mathcal{S}} dX_t(z_1) = 1 \quad \text{with } z_1 = (s, 0, 0, \eta_1, 0, 0) \quad (3-2)$$

must hold since we have normalized the cohort size of initial newborns to be unity. In the following, we will set $\mathcal{Z} = \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$ for the sake of simplification and omit the time index t and the state index z_j for every variable whenever possible. Agents are then only distinguished according to their age j .

3.1.2 Budget constraints and bequests

The budget constraint is defined by

$$a_{j+1} = a_j(1 + r) + y_j + p_j + b_j - \tau \min[w_j, 2\bar{w}] - T(y_j, p_j, ra_j) - (1 + \tau_c)c_j \quad (3-3)$$

with $a_1 = a_{J+1} = 0$ and $a_j \geq 0$ due to borrowing constraints. In addition to interest income from savings ra_j , households receive gross labor income $y_j = w(1 - \ell_j)e_j$ during their working period as well as public pensions p_j during retirement. As time endowment is normalized to one, ℓ_j defines leisure consumption and w the wage rate for effective labor. $e_j = e(z_j)$ denotes labor productivity that depends on age, skill level and labor market shock η_j . At specific ages households additionally receive accidental bequests b_j . Households have to pay social security contributions and taxes on labor, pension and interest income. The tax function $T(\cdot)$ will be

explained in more detail below. Due to a contribution ceiling which amounts to the double of average labor income \bar{w} , the contribution rate τ is not applied to income above the ceiling. Finally, the price of consumption good c_j includes consumption taxes τ_c . Of course, leisure can only be consumed up to the time endowment, i.e. $\ell_j \leq 1$.

Our model abstracts from annuity markets. Consequently, private assets of all agents who died are aggregated and then distributed equally among all working age cohorts younger than the earliest possible age of retirement.³ Consequently,

$$b_{jt+1}(z_j) = \frac{\Gamma}{(1+n_{t+1})(1+\lambda)} \sum_{i=1}^J \int_{\mathcal{Z}} (1-\psi_{i+1}^s)(1+r_{t+1})a_{i+1}(z_i) dX_t(z_i), \quad (3-4)$$

where Γ is the inverse of the sum of the working population below the earliest age of retirement and λ measures technological progress.

3.1.3 Individual preferences and the decision to retire

Our model assumes a preference structure that is represented by a time-separable, nested CES utility function. Individual period utility $u(c_j, \ell_j)$ depends on consumption of goods c_j and leisure ℓ_j . Households maximize intertemporal utility by taking into account the budget constraint (3-3). Technically, this decision problem is solved recursively. Consumption and leisure are chosen in order to maximize the utility function

$$V(z_j) = \max_{c_j, \ell_j} \left\{ u(c_j, \ell_j) + \beta \psi_{j+1}^s \left[(1 - \pi_{j+1,s}^d) \int_{\mathcal{E}} \left((1 - o_{j+1})V(z_{j+1}^w) + o_{j+1}V(z_{j+1}^r) \right) \pi^s(\eta_{j+1}|\eta_j) d\eta_{j+1} + \pi_{j+1,s}^d V(z_{j+1}^d) \right] \right\}, \quad (3-5)$$

with the terminal condition $V(z_{J+1}) = 0$. β defines the time discount factor and $\ell_j = 1$, if the household is already retired. We assume productivity shocks to be independent across individuals and to be identically distributed across individuals of a specific skill level. They follow a time and age independent Markov process, the conditional distribution of which is given by $\pi(\eta_{j+1}|\eta_j)$. As with productivity shocks, disability risk depends on individual skill

³Alternative bequest distributions have been explored but they don't alter the qualitative results of the policy experiments.

level. At each age j during the life-cycle an agent may get a disability or bad health shock with the probability π_j^d . Having received this shock, household's status changes to $d_j = 1$ and $o_j = 1$, since he will not be able to generate labor market income anymore. A disability pension system described in more detail below will care for the sustenance of disabled agents. In order to account for utility costs of this bad health shock, we restrict individual leisure consumption to a value of $h < 1$ that is lower than the maximum time endowment. This reflects the time cost of health care, e.g. visiting a doctor, or the utility costs of a reduced quality of life.

The three different combinations for z_{j+1} define the states in which the agent is still working in the next period, in which he chooses to retire and in which he receives a disability shock, i.e.

$$\begin{aligned} z_{j+1}^w &= (s, a_{j+1}, ep_{j+1}, \eta_{j+1}, 0, 0), \\ z_{j+1}^r &= (s, a_{j+1}, ep_{j+1}, 0, 0, 1), \\ z_{j+1}^d &= (s, a_{j+1}, ep_{j+1}, 0, 1, 1). \end{aligned}$$

At the beginning of each year of the retirement window between ages 60 and 70, households have to decide whether to retire or not, i.e. change their status from $o_j = 0$ to $o_j = 1$. Similar to Sánchez-Martín (2010) we assume retirement to be a one-time, irreversible decision which is derived from a comparison of utilities. Let $V(z_{j+1}^w)$ and $V(z_{j+1}^r)$ denote utilities from being in the labor force and being retired at age $j + 1$. Consequently we derive

$$\left[\frac{V(z_{j+1}^r)}{V(z_{j+1}^w)} \right]^{\frac{1}{1-\frac{1}{\gamma}}} - 1 + v(\eta_{j+1})$$

the consumption equivalent variation of retiring, where $v(\eta_{j+1}) \sim N(\mu_\eta, \sigma_\eta^2)$ captures additional non-pecuniary (i.e. psychological) benefits or costs from retirement which are not observed by the model. The individual taste parameter $v(\eta_{j+1})$ is related to own labor market status η_{j+1} , so that bad labor market shocks additionally give rise to earlier retirement beyond the incentives induced by low labor income.

Since we assume that those costs or benefits are normally distributed for each productivity shock, we can – due to the law of large numbers – compute the fraction of households that

decide to retire from

$$P \left(\left\{ \left[\frac{V(z_{j+1}^r)}{V(z_{j+1}^w)} \right]^{\frac{1}{1-\frac{1}{\gamma}}} - 1 + v(\eta_{j+1}) \right\} \right) = \Phi_{\mu_\eta, \sigma_\eta^2} \left(\left[\frac{V(z_{j+1}^r)}{V(z_{j+1}^w)} \right]^{\frac{1}{1-\frac{1}{\gamma}}} - 1 \right)$$

where $\Phi_{\mu_\eta, \sigma_\eta^2}(\cdot)$ is the distribution function of the normal distribution.

3.1.4 The production side

We let the production technology in our model be represented by a Cobb-Douglas production function $Y = \theta K^\epsilon L^{1-\epsilon}$, where K measures aggregate capital and L aggregate labor in efficiency units. The parameter ϵ denotes the share of capital in production and θ is a technology parameter. Capital depreciates at a constant rate δ_k and firms have to pay corporate taxes $T_k = \tau_k [Y - wL - \delta_k K]$, where the time-invariant corporate tax rate τ_k is applied to the output net of labor costs and depreciation. Firms maximize profits renting capital and hiring labor from households, so that net marginal products equal r the interest rate for capital and w the wage rate for effective labor. Finally, in order to account for technological progress, we follow Kotlikoff, Smetters and Walliser (2007) and assume time augmenting technological change.⁴ Consequently, individual time endowment increases by λ for any individual from period to period.

3.1.5 The government sector

3.1.5.1 The tax system

Our model distinguishes between the tax system and the pension system. In each period t the government issues new debt $(1 + n_{t+1})(1 + \lambda)B_{G,t+1} - B_{G,t}$ and collects taxes from households and firms in order to finance general government expenditure G_t , which is fixed per capita, as well as interest payments on its debt, i.e.

$$(1 + n_{t+1})(1 + \lambda)B_{G,t+1} - B_{G,t} + T_{y,t} + T_{k,t} + \tau_{ct}C_t = G_t + r_t B_{G,t}, \quad (3-6)$$

⁴Note that, due to the utility function not being of Cobb-Douglas type, we can not assume labor augmenting technological change, since this would not be consistent with a balanced growth path.

where revenues of income taxation are computed from

$$T_{y,t} = \sum_{j=1}^J \int_{\mathcal{Z}} T(y_j^l(z_j), y_j^r(z_j)) dX_t(z_j)$$

and C_t defines aggregate consumption (see (3-14)). The intertemporal budget is balanced by consumption taxation.

Our model takes into account the transition towards deferred taxation of pension benefits in Germany introduced in 2005. Consequently, taxable labor income y_j^l is computed from gross labor income net of (a fraction κ_1 of) pension contributions and a work related allowance $d(w_j)$ and – after retirement – (a fraction κ_2 of) public pensions. With respect to taxable interest income we apply as an approximation to a variety of saving incentives a fixed saving allowance d_s , so that

$$y_j^l = \max[w_j - \kappa_1 \tau \min[w_j, 2\bar{w}] - d(w_j); 0] + \kappa_2 p_j \quad \text{and} \quad y_j^r = \max[ra_j - d_s, 0].$$

Given taxable income, we apply the German progressive tax code of 2005 to labor income and assume that all households are married couples (i.e. full income splitting). Interest income, however, is taxed at a constant rate τ^r which reflects the flat capital income tax recently introduced in Germany. Finally, an additional surcharge of 5.5 percent is applied to the tax burden.⁵ Consequently,

$$T(y_j^l, y_j^r) = 1.055 \times (2 \times T05(y_j^l/2) + \tau_r y_j^r),$$

where $T05(\cdot)$ denotes the marginal tax rate schedule of year 2005.

3.1.5.2 The pension system

In each period t , the pension system pays old-age and disability benefits and collects payroll contributions from labor income below the contribution ceiling $2\bar{w}_t$. Individual pension benefits p_j of a retiree of age $j \geq j_R$ in a specific year are computed from the product of the adjustment factor $\nu(j_R)$ which depends on the individual retirement age j_R , earning points ep_{j_R} he has

⁵The so-called "solidarity" surcharge was introduced in 1990 in order to finance the burden of the German unification.

accumulated until retirement age (see equation (3-9) below) and the actual pension amount (*APA*) per earning point:

$$p_j = \nu(j_R) \times ep_{j_R} \times APA. \quad (3-7)$$

When a worker retires early, i.e. before the so-called normal retirement age (NRA), his earning points will be adjusted in order to account for the prolongation of the retirement period. According to German law, they are reduced by 3.6 percent for any year the agent retires before the normal retirement age. However, as will be discussed below, the model abstracts from increases in pensions due to delayed retirement, so that

$$\nu(j_R) = \begin{cases} 1 - (\text{NRA} - j_R) \times 0.036, & j_R < \text{NRA} \\ 1, & j_R \geq \text{NRA}. \end{cases} \quad (3-8)$$

A special rule applies to individuals who become disabled – and therefore have to exit the labor force – before the age of 60 and receive disability payments from the pension system. In order to derive their pension payments in line with German law, it is assumed that they had worked up to the age of 60 with their average productivity. Hence, their pension is adjusted in order to correct for the missing years of work.

Accumulated earning points of the pension system depend on the relative income position w_j/\bar{w} of a worker. Since the contribution base ceiling is fixed at the double of average income \bar{w} , maximum earning points collected per year are 2. Therefore, earning points accumulate according to

$$ep_{j+1} = ep_j + \min[w_j/\bar{w}; 2], \quad (3-9)$$

where $ep_1 = 0$.

Finally, the actual pension amount APA_t of a specific year t is adjusted according to

$$APA_t = APA_{t-1} \times \frac{w_{t-1}L_{t-1}(1 - \tau_{t-1}^p - \tau_{t-1})}{w_{t-2}L_{t-2}(1 - \tau_{t-2}^p - \tau_{t-2})} \times \left\{ 1 + 0.25 \times \left(1 - \frac{PR_{t-1}}{PR_{t-2}} \right) \right\}. \quad (3-10)$$

Equation (3-10) reflects the central elements of the adjustment formula which was introduced

by the pension reforms 2001 and 2004. Since then, changes in the actual pension amount are related to lagged changes of an artificial income concept which is computed from gross labor earnings net of fictional contributions τ^p (which amount to 3 percent before and 4 percent after 2008) to the private pension scheme and actual contributions to the public scheme. The last part of (3-10) reflects the sustainability factor where PR defines the pensioners ratio which measures the ratio of retired to working households of a specific year.⁶ Since this pensioners ratio will increase in the future, the adjustment factor will decrease future benefits. However, the impact of the rising dependency ratio is dampened by the weight 0.25. Note that any delay in retirement induced by the reform of 2007 will dampen the sustainability factor and increase benefits of already retired households.

The budget of the pension system must be balanced in every period by adjusting the contribution rate, so that

$$\sum_{j=60}^J \int_{\mathcal{Z}} p_j(z_j) dX_t(z_j) = \tau_t \sum_{j=1}^{69} \int_{\mathcal{Z}} \min[w_j(z_j); 2\bar{w}] dX_t(z_j). \quad (3-11)$$

3.1.6 Welfare and efficiency calculation

In order to compare welfare for a specific individual before and after the reform, we follow Auerbach and Kotlikoff (1987, 87) and compute the proportional increase (or decrease) in consumption and leisure ϕ which would make an agent in the baseline path as well off as after the reform. We can compare all cohorts living in the reform year $t = 1$ and all newborn cohorts along the transition path before and after the reform since they have identical individual state variables. Due to the homogeneity of the utility function (3-16) and (3-5) the necessary increase (or decrease) in percent of resources is

$$\phi_t(z_j) = \left\{ \left[\frac{V_t(z_j^1)}{V_t(z_j^0)} \right]^{\frac{1}{1-\frac{1}{\gamma}}} - 1 \right\} \times 100,$$

⁶Strictly speaking, the pensioners ratio is computed in practice from the standardized numbers of “equivalence pensioners” and “equivalence contributors” derived from (fictive) standard pensions and average earnings.

where z_j^0 and z_j^1 indicate that utility of the specific person is measured before and after the reform, respectively. Consequently, a value of $\phi_t(z_j) = 1.0$ implies that this agent would need one percent more initial endowment in the baseline path to attain the utility level he receives after the policy reform.

In order to assess aggregate efficiency consequences, we introduce a Lump-Sum Redistribution Authority (LSRA) in the spirit of Auerbach and Kotlikoff (1987, p. 62f.) as well as Fehr (2000) or Nishiyama and Smetters (2007) in a separate simulation. The LSRA treats those cohorts already existing in the initial year 2008 and newborn cohorts differently. To already existing cohorts it pays a lump-sum transfer (or levies a lump-sum tax) $v_{j,1}(z_j), j > 1$ to bring their expected utility level after the reform back to the level of the initial equilibrium $V_1(z_j^0)$. Since utility depends on age and state, these transfers (or taxes) have to be computed for every agent in the first year of the transition. Consequently, after compensation, their relative welfare change is $\phi^c(z_j) = 0.0$. Furthermore, those who enter the labor market in period $t \geq 1$ of the transition receive a transfer $v_{1,t}(z_1, \phi^{c*}(z_1))$ which guarantees them a (compensated) relative consumption change $\phi^{c*}(z_1)$ which is identical for all newborn future cohorts. Note that the transfers $v_{1,t}(z_1, \phi^{c*}(z_1))$ may differ among future cohorts but the relative utility change $\phi^{c*}(z_1)$ is identical for all. This utility change is determined by requiring that the present value of all LSRA transfers is zero:

$$\sum_{j=2}^J \int_{\mathcal{Z}} v_{j,1}(z_j) dX_1(z_j) + \sum_{t=1}^{\infty} v_{1,t}(z_1, \phi^{c*}(z_1)) \prod_{s=2}^t (1 + n_s) (1 + \lambda) (1 + r_s)^{-1} = 0.$$

In the first period of the transition the LSRA builds up debt (or assets) from

$$(1 + n_2)(1 + \lambda)B_{RA,2} = \sum_{j=1}^J \int_{\mathcal{Z}} v_{j,1}(z_j) dX_1(z_j)$$

which has to be adjusted in each future period according to

$$(1 + n_{t+1})(1 + \lambda)B_{RA,t+1} = (1 + r_t)B_{RA,t} - v_{1,t}(z_1). \quad (3-12)$$

Of course, LSRA assets are also included in the asset market equilibrium condition (3-15).

If $\phi^{c^*}(z_1) > 0$ ($\phi^{c^*}(z_1) < 0$), all households in period $t = 1$ who lived in the previous period would be as well off as before the reform and all current and future newborn households would be strictly better (worse) off. Hence, the new policy is Pareto improving (inferior) after lump-sum redistributions.

3.1.7 Equilibrium conditions

Given a specific fiscal policy, an equilibrium path of the economy has to solve the households decision problems (3-5), reflect competitive factor prices and balance aggregate inheritances with unintended bequests. Furthermore, in the closed economy aggregation holds,

$$L_t = \sum_{j=1}^J \int_{\mathcal{Z}} (1 - \ell_j(z_j)) e_j dX_t(z_j), \quad (3-13)$$

$$C_t = \sum_{j=1}^J \int_{\mathcal{Z}} c_j(z_j) dX_t(z_j), \quad (3-14)$$

$$K_t = \sum_{j=1}^J \int_{\mathcal{Z}} a_j(z_j) dX_t(z_j) - B_{G,t} - B_{RA,t}, \quad (3-15)$$

the budgets of the government (3-6), the pension system (3-11) and the redistribution authority (3-12) are balanced and the goods market clears in every period, i.e.

$$Y_t = C_t + G_t + (1 + n_{t+1})(1 + \lambda)K_{t+1} - (1 - \delta_k)K_t.$$

The computational method to solve the model numerically follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for aggregate variables, bequest distribution and policy parameters. Then we compute factor prices, individual decision rules and value functions which involves discretization of the state space and the use of multidimensional spline interpolation. Next we obtain the distribution of households and aggregate assets, labor supply and consumption as well as payroll and consumption taxes in order to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values of macro variables and policy parameters have sufficiently converged.

3.2 Calibration

Since we use realistic demographic forecasts in our model, our base year 2008 will not be a long-run equilibrium and the benchmark simulation will be a baseline path simulated under demographic transition and including recent reforms of the tax and pension system. In the following, we will discuss how we calibrated the initial year of our simulation model and how our baseline path will be determined. Then we move on to policy reforms and their macroeconomic, welfare and efficiency consequences.

3.2.1 Demographic projections

In our model, one period covers one year. We assume agents to start their economically relevant life at the age of 20 and to live up to a maximum of 100 years. In order to get a reasonable classification of skills, we use the International Standard Classification of Education (ISCED) of the UNESCO. We thereby merge levels 0 to 2 (primary and lower secondary education), levels 3 and 4 (higher secondary education) and levels 5 and 6 (tertiary education) in order to receive 3 skill levels, i.e. $S = \{1, 2, 3\}$. The household shares in the different skill classes are calculated using data from the German Socio-Economic Panel (SOEP), a description of which can be found in Wagner, Frick and Schupp (2007). In this representative data set, low-, medium- and high-skilled individuals represent 26, 55 and 19 percent of the population, respectively. Survival probabilities $\psi_{j,2}$ for the middle skill class are taken from the 2000 Life Tables for Germany reported in Bomsdorf (2003). Taking into account the positive correlation between skill level and life expectancy, we compute probabilities for the low and the high skill class from $\psi_{j,2}$ in a way that life expectancy between those two differs by 5 years, see von Gaudecker and Scholz (2007). Consequently, initial life expectancies are 77.1, 79.6 and 82.1 for the three different skill classes, respectively.

To account for a reasonable demographic transition, we set the population structure in 2008 to the one observed in Germany. Taking one of the "benchmark scenarios" from SB(2009), we assume the total fertility rate to remain constant at 1.4 children per woman, life expectancy to increase linearly for any skill class by 7.3 years until 2060 and net migration to rise gradually from 50.000 to 100.000 until 2014. For the sake of simplicity, we assume that all migrants are

age 20 when they enter the economy and have the same skill composition as natives. In order to converge to a stable population and be able to compute a long-run equilibrium, we set the population growth rate n_t to 0 and keep life-expectancy and migration unchanged from 2060 onwards. Figure 3-1 reports the resulting population structure from 2008 to 2100.

FIGURE 3-1 HERE.

As can be seen, we start with a population size of 82 million in 2008 which declines to 65.6 million in 2060. This is quite close to the numbers reported in SB (2009) who project a population size of 64.7 million. With the rise in life expectancy and low fertility rates, the old age dependency ratio – which is defined as the number of individuals aged 60 or older over the number of people between ages 20 and 59 – increases from currently 46.1 percent to 89.4 in 2060.

3.2.2 Labor productivity, idiosyncratic risk and the distribution of incomes

Following Love (2007), we let log-productivity for an individual of skill class s evolve over the life-cycle according to

$$\log e_j = \varsigma_0 + \varsigma_1 \cdot j + \varsigma_2 \cdot j^2/100 + \eta_j$$

where η_j follows an AR(1) process of the form

$$\eta_j = \varrho\eta_{j-1} + \varepsilon_j \quad \text{with} \quad \varepsilon_j \sim N(0, \sigma_\varepsilon^2).$$

In order to estimate productivity profiles for the three skill levels, we use inflated labor income data of full-time primary household earners from the SOEP. Our unbalanced data set covers workers between ages 20 to 60 of the years 1984 to 2006 that were again divided into three skill classes according to the ISCED standard. This leads us to a total of 83893 observations with 11789, 55015, and 17089 in the three different classes, respectively. With this data set, we estimate the above equation separately for any skill class s , using some dummy variables reflecting job type and family status as well as an individual persistent effect. The results of the estimation process can be found in Table 3-1, standard errors are reported in parenthesis.

Table 3-1: Parameter estimates for individual productivity

	Low-skilled	Middle-skilled	High-skilled
Intercept and type ς_0	9.6207 (0.2662)	9.4190 (0.1494)	8.6649 (0.3116)
age term ς_1	0.0437 (0.0041)	0.0579 (0.0025)	0.1025 (0.0064)
age ² term ς_2	-0.0500 (0.0052)	-0.0649 (0.0031)	-0.1090 (0.0074)
AR(1) correlation ϱ	0.7244 (0.0119)	0.7826 (0.0046)	0.7770 (0.0088)
transitory variance σ_ε^2	0.0646 (0.0056)	0.0737 (0.0039)	0.0790 (0.0076)

There are two things to notice. First, we find a strong AR(1) correlation of around 0.8 for the error term η_j , which lies in the range of typical values for these types of models, see e.g. Love (2007) or İmrohoroğlu and Kitao (2009). Second, except for high-skilled, we see a small persistent variance, which means that our groups are strongly homogeneous. In the highest educational group, however, there is a certain chance of climbing up into the area of extraordinary high salaries. This makes the group somewhat more heterogeneous and explains a higher variance of the individual effect.

In our model, we use the estimated profiles in between the ages 20 and 65. Note that, with rising age, the estimates become more and more biased upwards, since individuals with low productivity, e.g. due to bad health, might already have become retired. Hence, from the age of 65 onwards, we assume in line with Eisensee (2005), that productivity depreciates quadratically until it finally reaches zero at age 75. The productivity profiles can be seen in Figure 3-2.

FIGURE 3-2 HERE.

In terms of income uncertainty, we discretize the estimated AR(1) process, using the procedure described in Tauchen and Hussey (1991) with 3 approximation points, since this algorithm delivers the most accurate results in terms of income distribution, see Kindermann (2010). Table 3-2 compares some model outcomes with income and asset inequality measures in Germany for 2007 taken from SVR (2009).

Table 3-2: Measures of income inequality

		Percentage share of		Gini index
		Lowest 10 %	Highest 10 %	
Net Income	Model	2.8	22.8	0.283
	Data*	3.6	24.0	0.290
Assets	Model	0.0	38.9	0.569
	Data*	-1.1	61.0	0.799

*Source: SVR (2009)

We find quite a good match of both the lowest and highest percentile of the income distribution as well as the Gini index. Of course, since individuals are not allowed to run into debt and we do not explicitly account for very high income earners and self-employed, the asset distribution is more equal in our model than in reality.

3.2.3 Disability risk

Starting from age 30, we assume the probabilities of becoming disabled to be positive. Hagen et al. (2010) report disability risk for workers with different skill levels between the ages 30 and 59. They find that the probability of becoming disabled increases exponentially with age. Consequently, we also assume exponential growth of disability risk throughout the working life and extrapolate the probabilities mentioned above from age 59 up to age 70. As consequence of disability, we force workers to retire and let them suffer from a reduction in their time endowment of 20 percent, i.e. $h = 0.8$. This reflects the time cost of health care (e.g. visiting a doctor) and yields realistic utility costs of a reduced quality of life, see below.

With this specification, the higher an agent's skill level, the higher his losses from disability. This is due to the fact that the higher skilled face the steeper increase in labor productivity throughout the life cycle, see Figure 3-2. Consequently, their forgone earnings from disability are especially large at the beginning of the life cycle, while at older ages this difference shrinks noticeably. Furthermore, utility costs are lowest if individuals become disabled around the age of 50. This is due to the fact that average productivity is the highest at this point in the life cycle. Since earning points of a disability pensioner are calculated by assuming he had worked the

remaining years until age 60 with his average productivity, benefits from this disability subsidy reach their maximum at age 50 and decline with a decreasing labor productivity afterwards. Overall, the maximum average welfare loss from disability amounts to roughly 30 percent. Given the results from Torrance, Boyle and Horwood (1982), who estimate direct utility losses from disabilities between 30 and 70 percent, we feel that our approach is quite conservative.

3.2.4 Preferences and the decision to retire

We let individual preferences over consumption and leisure be represented by the instantaneous CRRA utility function

$$u(c_j, l_j) = \frac{1}{1 - \frac{1}{\gamma}} \left\{ c_j^{1 - \frac{1}{\rho}} + \alpha l_j^{1 - \frac{1}{\rho}} \right\}^{\frac{1 - \frac{1}{\gamma}}{1 - \frac{1}{\rho}}}, \quad (3-16)$$

where ρ denotes the intra-temporal elasticity between consumption and leisure and α is a taste parameter for leisure consumption. γ represents the inter-temporal elasticity of substitution between consumption in different years. In order to calibrate the parameters of the utility function we first set γ at 0.5, which is in the range of commonly used parameters in these types of models, see İmrohoroğlu and Kitao (2009).⁷ We then chose $\rho = 0.6$ in order to obtain realistic labor supply elasticities. In order to set average hours worked in the economy at 0.4, which implies a 40 hours work week length, we let $\alpha = 1.6$. Finally, we chose a value of 0.985 for β to obtain a capital to output ratio of 3.5, which is close to the one observed in Germany in 2008. We also calculate wage elasticities of labor supply for individuals between ages 20 and 60. The values of the uncompensated elasticities are around zero and increasing with age, which corresponds closely with Fenge, Übelmesser and Werding (2006). The compensated elasticities are also increasing with age around a value of 0.2.

The German pension system actually has two normal retirement ages, i.e. the first age at which one will receive the full pension payment without adjustment. The regular normal retirement age is 65. However, a reduced age of 63 applies to both disability pensioners and the unemployed. Since we explicitly account for disability risk, normal retirement age is set to 63 for

⁷Models that abstract from precautionary saving motives typically apply lower values for the intertemporal elasticity of substitution.

these individuals. Note that in consequence, a disabled worker who has to retire before age 60, will have his earning points adjusted as if he had worked until age 60, but will also face an adjustment factor of $(63 - 60) \cdot 3.6 = 10.8$ percent. In addition, we interpret our worst labor market shock as old-age unemployment and let individuals with this shock also retire with a normal retirement of 63. Furthermore, the age at which individuals can make their first retirement choice differs across different groups. Whereas disabled individuals always retire immediately, the unemployed may retire from age 60 onwards while regular workers may start retiring from 63. As mentioned above, psychological costs of working v in our model depend on the own labor market status. With these cost parameters, we target a good match of the retirement pattern currently observed in Germany. Nevertheless, we also aim at minimizing the influence of this (exogenous) parameter on the decision to retire. Hence, we let v have an expected value of 1.17 percent, a standard deviation of 1.27 percent and assume a correlation between v and η_j of -0.9 , i.e. the higher labor market income, the lower utility costs from working. This results in the retirement pattern reported in the left part of Figure 3-3. Due to the very detailed modeling of the German pension law, we find a good match of the model generated retirement pattern with German data. Obviously, workers retiring before age 60 have to be disabled. After age 60 we see three major peaks at ages 60, 63 and 65. The first of these is due to retirement of the unemployed. The second peak, on the other hand, is formed by individuals having had a lot of luck during their time of work and having accumulated a lot of assets and earnings points. Since those will not work anymore due to income effects, they may retire early. The last peak captures all regular old-age retirees.

FIGURE 3-3 HERE.

The right part of Figure 3-3 shows the retirement pattern for individuals of different skill classes. Since the risk of getting disabled decreases with skill level, the lower skilled tend to retire earlier than the higher skilled.

3.2.5 Technology

IdW (2009) reports a wage share in production of 65.2 percent for Germany in 2008. Since the wage share in our model is given by $1 - \epsilon$, we set ϵ at 0.35. In addition, we choose a value

of 1.3 for the technology parameter θ in order to normalize the wage rate for effective labor to unity. We let the depreciation rate δ_k on capital be 4.2 percent. This guarantees investment to amount to 20.3 percent of GDP, which is slightly higher than the value of 19.3 reported in IdW (2009) for Germany. Finally, we assume a growth rate of 1.3 percent for individual time endowment, which is in line with the long run average growth rate for Germany reported in Erber and Fritsche (2009).

3.2.6 Government policy

The tax system Government tax policy in our model reflects quite well the German tax system. Specifically, we set the debt to output ratio at 60 percent and fix the consumption tax rate at 17 percent, which guarantees a consumption tax revenue to output share of 9.9 percent. This share is slightly lower than the value of 10.7 percent reported in IdW (2009). We apply the German income tax code of the year 2005 to labor and pension income. Consequently, when taxable income passes the basic allowance level of roughly 8.000 Euro, the marginal tax rate jumps to 17 percent and then further rises up to 42 percent when taxable income is beyond 52.000 Euro. In addition we tax returns from savings linearly at the rate 26.4 percent. This reflects the recent reform of capital income taxation in Germany. Finally, we set the corporate tax rate τ_k at 15 percent which yields a revenue to output ratio of 3 percent that is slightly higher than the value of 2.1 percent reported in IdW (2009).

The pension system We provide more detail on the pension system, since it is the focus of this paper. As already mentioned above, we set normal retirement ages, ages of first retirement choice and the adjustment factor for early retirement according to German pension law. We also use realistic disability risk processes derived from Hagen et al. (2010). Finally we set the pension contribution rate to 19.9 percent, which since 2007 is the legal contribution rate in Germany. This leads us to a gross replacement rate of 48 percent, which is 5 percentage points above the one reported for Germany in OECD (2009, p. 201), and an overall amount of pension payments of 12 percent of GDP. Thereby, 1.5 percent of GDP are spent on disability pensions, a value slightly higher than the amount of 0.9 percent reported for Germany by IdW (2009). As already discussed before, changes in the actual pension amount are derived from the evolution

of aggregate household income corrected for changes in the old age dependency ratio.

3.2.7 The baseline path

Except for the demographic projections, all the figures above were calculated to calibrate our base year 2008. Table 3-3 again summarizes our calibrated parameters and the respective calibration targets.

Table 3-3: Calibration targets

Parameter		Value	Target
<i>Demographics</i>			
Population growth	n_t		Population projections by SB (2009)
Survival probabilities	ψ_j		Bomsdorf (2003), SB(2009), von Gaudecker and Scholz (2007)
Skill distribution	ϖ_s		estimated from SOEP data
<i>Labor productivity and disability risk</i>			
Labor productivity	e_j, η_j		estimated from SOEP data
Disability risk	π_j^d		Hagen et al. (2010)
Time loss from disability	h	0.80	welfare loss from disability
<i>Preferences and retirement</i>			
Inter-temporal elasticity of substitution	γ	0.50	İmrohoroğlu and Kitao (2009)
Intra-temporal elasticity of substitution	ρ	0.60	labor supply elasticities
Leisure preference	α	1.60	average hours worked 0.4
Time discount factor	β	0.985	capital output ratio 3.5
psychological costs of working	μ_η	1.17	retirement pattern
	σ_η^2	1.27	
<i>Technology and government policy</i>			
Capital share in production	ϵ	0.35	IdW (2009)
Technology parameter	θ		wage rate for effective labor of 1
Depreciation of capital	δ_k	0.042	investment to GDP ratio 19.3%
Debt to GDP ratio		0.60	target value issued by government
Consumption tax rate	τ_c	0.17	revenue to output share 10.7%
Income tax code			German tax law
Corporate tax rate	τ_k	0.15	revenue to output share 2.1%
Technological progress	λ	0.013	Erber and Fritsche (2009)
Pension system design			German pension law

However, since we assume a demographic transition, we need a complete baseline scenario from 2008 to a long-run equilibrium as basis for our computational experiment. In order to calculate this baseline path, we have to make some specific assumptions about future public policy. More specifically, our baseline path includes the transition towards front-loaded taxation

of pension benefits as implemented in year 2005. As a consequence, in 2008, 66 percent of pension contributions are exempt from tax, while 56 percent of received pension payments have to be taxed. These rates gradually increase over time until in 2040, pension contributions are completely deductible and benefits are fully taxed. With respect to the future development of the pension contribution rate the government has issued the targets of 20 percent for 2020 and 22 percent for 2030. While the increase of the contribution rate is already dampened due to previous reforms, the medium target for 2030 could only be matched with additional adjustments of the actual pension amount. Finally, the baseline path abstracts from the recently implemented increases of the normal retirement ages of the cohorts born 1947 and later in order to provide a detailed analysis of this reform in the simulation part. Table 3-4 reports our simulated baseline path from 2008 until 2060.

Table 3-4: Baseline path of the economy

Year	2008	2020	2030	2040	2050	2060
<i>Macroeconomic aggregates</i>						
Output	100.0	112.5	119.2	125.7	133.8	142.4
Employment	100.0	109.4	113.5	119.4	127.5	135.6
Capital	100.0	118.3	130.7	138.5	146.5	156.0
<i>Prices</i>						
Wage	100.0	102.8	105.1	105.3	105.0	105.0
Interest rate	4.9	4.5	4.2	4.1	4.2	4.2
Consumption tax rate	17.0	20.1	22.6	24.3	25.2	25.3
<i>Pension system</i>						
Mean retirement age	59.8	60.6	61.4	60.6	60.9	60.9
Replacement rate	48.0	42.7	38.3	37.2	37.0	36.8
Pension expenditure	12.0	12.7	14.2	15.2	15.5	15.8
Pension contribution rate	19.9	21.1	23.7	25.3	25.9	26.3
<i>Poverty measures</i>						
Elderly poverty rate	5.0	8.0	10.4	11.0	11.0	10.5
Elderly poverty risk	17.4	19.5	19.2	19.5	19.7	19.5

There are two main driving forces determining these results. On the one hand, the exogenous rise in labor augmenting productivity increases effective employment despite the decrease in the working age population. On the other hand, the strong increase in the old age dependency ratio induced by a rise in longevity and low fertility rates causes a reduction in pension benefit

levels. As a consequence, aggregate savings and therefore capital increase much stronger than employment, so that wages increase throughout the transition while the interest rate falls. Due to population aging the size of the work force decreases much faster than the size of the overall population. In addition, public goods provision is constant per capita. Consequently, revenues from income taxation decline much faster than public expenditure which results in increasing consumption taxes. Rising longevity may be the main reason for the slight increase in the mean retirement age.⁸ Although the replacement rate declines throughout the transition, the budget of the pension system still has to rise due to the increase in pensioners. Consequently, the pension contribution rate has to be successively adjusted until it reaches a value of 26.3 percent in 2060. Note that due to the additional reduction in pension benefits we are nearly able to match the targets for the contribution rate of 20 percent in 2020 and 22 percent in 2030 issued by the German government.

Finally, we report some poverty measures amongst the elderly in our baseline scenario. According to SVR (2007), a household is considered to be poor if its after tax income lies below 40 percent of the median income in the population. In addition, according to the definition of the European Union, one is at risk of poverty if income is not higher than 60 percent of median income. In the last part of Table 3-4, we therefore report poverty rates and rates of poverty risk for pensioners. We find that about 5 percent of retirees are poor and 17.4 are at the risk of being so. These numbers are higher than the 2.4 percent and 15.0 percent reported for Germany in SVR (2007), but our model does not account for the means-tested social assistance system which helps the poor in Germany. Therefore, we think that these two measures still are a good indicator for the evolution of old-age income in our model. Due to the decline of the replacement rate throughout the transition, elderly become increasingly poor and the poverty rate more than doubles until 2040.

3.3 Simulation of policy reforms

In this section we apply the model to analyze a range of policy reforms that are intended to increase the retirement age, to alter progressivity and to increase the funding of the system. In

⁸But this is not so clear since our model abstracts from annuity markets. As shown in Kalemli-Ozcan and Weil (2010), the effect of decreasing mortality rates on the timing of retirement is ambiguous when there are unintended bequests.

the next subsection we will discuss the modeling of each reform in detail. Then we present the macroeconomic impact of these reforms before we move on to the welfare effects. Finally we present some sensitivity analysis.

3.3.1 Increasing retirement age, progressivity and funding of pensions

As already explained above, the baseline simulation assumes that the two normal retirement ages remain constant despite the significant increase in life expectancy. Of course, later retirement is one option to combat population aging and Germany has discussed various measures to achieve that. The first reform therefore considers the recent pension reform of 2007 which increases the normal retirement age from currently age 65 to age 67 and the reduced normal retirement age from currently 63 to 65. More specifically, starting in 2012 the two normal retirement ages for the cohort born in year 1947 increase by one month. For the cohort born in 1948, they increase by two months and so forth. For cohorts born in and after year 1959, the normal retirement ages increase by fourteen, sixteen, eighteen months and so on until, finally, cohorts born in and after 1964 all face an increase of two years. This initial reform is compared with an alternative package proposed by Berkel and Börsch-Supan (2004) or Queisser and Whitehouse (2006). While in the baseline path the adjustment factor (AF) per year of early retirement is 3.6 percent, the reform increases the adjustment factor to eventually 6 percent. Again, the increase is phased-in – as the NRA – between 2012 and 2029.

Next, we abstract from measures that alter retirement ages and consider two reforms which increase the progressivity of the pension system. In Germany two such policies are discussed at the moment. Since life expectancy and lifetime labor income are positively correlated, Breyer and Hupfeld (2009) propose a correction factor for higher life expectancy in order to achieve a greater distributional justice. Following their suggestion, we supplement the pension formula (4-21) by the factor $\frac{10}{5+5ep_{j_R}/j_R}$. Since in our model low income households (i.e. where $ep_{j_R}/j_R < 1$) also have a lower life expectancy, the correction factor increases their pension benefit. The opposite holds for high income households. The other option to increase the progressivity of the pension system is to increase the accumulation of earning points for lower incomes. More specifically, if labor income of a household is below 75 percent of average

income \bar{w} , the accumulation formula (3-9) changes to

$$ep_{j+1} = ep_j + \min[1.5w_j/\bar{w}; 0.75], \quad \text{if } w_j < 0.75\bar{w}.$$

The final set of simulations analyzes two alternative ways to move towards a funded pension system. In both cases we assume that at the beginning of 2009 the government announces a complete phase-out of pension benefits starting for the cohort born in year 1960. Whereas benefits of this cohort are reduced by 5 percent after retirement, the following cohort experiences a reduction of 10 percent and so on until the cohort born in 1980 receives no pension benefits any more. In order to balance the burden of privatization across the different cohorts, we build up pension debt in order to generate a constant reduction of the payroll tax rate for all future cohorts. Alternatively, the last simulation proposes a different financing mix where part of the privatization is financed by consumption taxes.

The next subsection discusses the macroeconomic effects of these policy packages. We then report the resulting welfare and efficiency effects and perform some sensitivity analysis.

3.3.2 Macroeconomic effects of pension reforms

At first we want to take a look at the macroeconomic implications of the different reforms considered. The first two columns of Table 3-5 report the changes in different variables when we increase the cost for earlier retirement. The increase in the normal retirement age as implemented by the pension reform 2007 increases the mean retirement age by 15.6 months in the long run.⁹ Since with unchanged retirement behavior the reform would reduce future pension benefits significantly, people delay retirement significantly on average, which in turn increases employment slightly by 0.2 percent. The small employment effect indicates a strong intertemporal substitution of labor supply. When households plan to work longer at old age, they reduce labor supply at younger ages.¹⁰ When they work less at younger ages, savings also decline temporarily which explains the short-run reduction in the capital stock. The increase of the mean

⁹In the following discussion, the notion "in the long run" refers to the equilibrium in year 2060 and not to the long run steady state equilibrium.

¹⁰This latter finding corresponds well with the recent discussion in İmrohoroğlu and Kitao (2009). They show that pension reforms have only minor effects on aggregate employment but they change the life-cycle profile of labor supply.

retirement age is roughly in line with the results from Mastrobuoni (2009), who finds evidence that the mean retirement age of the affected cohorts in the U.S. increases by about half as much as the increase in the NRA.¹¹ In the medium and long run, savings grow more than employment, so that wages increase slightly. Higher income tax revenues allow reductions in consumption taxes and the longer working phase allows reduced contribution rates.

Next, consider in the second column of Table 3-5, which examines the alternative reform where we increase the adjustment factor between 2012 and 2029 from 3.6% to 6%. As one can see, this has only a minor effect on the mean retirement age. Despite the small impact on retirement, contribution rates decrease almost as before, because of significant reductions in benefits. The latter induces an increase in private savings, so that capital accumulation rises much more than before. Since employment is only hardly affected, wages now increase stronger than before.

In the third and fourth column we show the results of introducing two reforms which increase the progressivity of the pension system. While the correction factor for life expectancy (LE) has a significant impact on mean retirement age, the upgrade of earnings points for low earnings has no effect on retirement behavior. Of course, in the first case mainly high-skilled households delay retirement, since they face lower retirement benefits. The transfer of resources from high towards low income households reduces aggregate employment and savings in both cases. As a consequence wages decrease, so that income tax revenues fall and the consumption tax rate has to increase. In the first case the impact on the consumption tax is stronger than in the second case. On the other hand, the increase of contribution rates is almost the same in both simulations.

The last part of Table 3-5 shows two simulations where the paygo pension system is completely eliminated. While the first reform uses payroll taxes to finance existing pension claims, the second one uses a mixture of consumption and payroll taxes. In order to distribute the burden equally across future generations, we assume a constant tax rate reduction in all periods after the announcement and balance the system by debt adjustment. Since individuals after the reform have to privately care for their old-age consumption, the savings level is much higher

¹¹Of course, one should not overemphasize these comparable findings, since the figures are derived empirically and in a different context. For example, in the U.S. system, the NRA increase from 65 to 67 reduces final benefits by 10 percent (Mastrobuoni, 2009).

than in the baseline scenario. In the second reform the increase in assets and therefore capital is much stronger, since higher consumption taxes raise the need for resources at retirement. More assets and therefore more interest income obviously result in higher income tax revenues which causes the consumption tax rate to decline. This effect is enforced by the fact that pension contributions are exempt from tax. As the payroll tax decreases by 6.0 percentage points, more income is taxed during the working periods on a progressive basis. Overall, the increase in income tax revenues causes a decline in the consumption tax rate of about 4.3 percentage points. In the second simulation, this effect is damped by the need for higher consumption taxes to finance existing pension claims. In turn, payroll taxes can be reduced further. With the decline in payroll and consumption taxes, aggregate employment is higher in the short-run compared to the baseline scenario. In the long run, however, employment decreases albeit a significant increase in retirement ages, which indicates a strong intertemporal substitution of labor supply and higher labor market distortion induced by a switch from front- to back-loaded income taxation.

3.3.3 Welfare and efficiency effects

So far, we have only considered the evolution of macroeconomic aggregates over time. However, we are also interested in the intra- and intergenerational welfare consequences of the considered reforms. Therefore Table 3-6 summarizes welfare consequences measured in equivalent variation for different cohorts. For agents already taking economic decisions in the reform year, we disentangle welfare effects in several ways. We report for agents already retired in the reform year average welfare changes by their status in the pension system, i.e. regular or disability pensioner. The working cohorts are, beneath explicitly accounting for welfare of the disabled, grouped by their productivity level. We thereby merge the lower one sixth and the higher one sixth of the overall productivity distribution in the states "low" and "high". The remaining part of the distribution is captured in state "medium".¹² For future generations, we apply the concept of ex ante welfare and therefore only report one aggregate number per cohort. The first two columns indicate birth year of the respective cohorts and their age in the reform year 2009.

¹²We chose exactly this partition of the work force due to computational reasons.

Table 3-5: Macroeconomic effects of pension reforms

	<i>Retirement age</i>		<i>Progressivity increase</i>		<i>Privatization</i>	
	<i>NRA67</i>	Adjustment factor 6%	Correction factor LE	EP upgrade	Payroll taxes	Consumption taxes
<i>Output</i>						
2020	0.1	0.2	-1.9	-0.8	0.9	1.3
2040	0.3	0.3	-2.5	-1.2	0.9	1.6
2060	0.3	0.3	-2.9	-1.6	1.5	2.5
<i>Capital stock</i>						
2020	-0.1	0.4	-1.9	-0.8	0.9	2.6
2040	0.5	0.9	-3.8	-2.2	1.9	5.3
2060	0.5	1.2	-5.0	-3.4	6.3	10.4
<i>Employment</i>						
2020	0.2	0.1	-1.9	-0.8	0.8	0.6
2040	0.1	-0.1	-1.8	-0.7	0.3	0.0
2060	0.2	-0.1	-1.8	-0.7	-0.6	-1.1
<i>Wage rate</i>						
2020	-0.1	0.1	0.1	0.0	-0.1	0.5
2040	0.1	0.3	-0.5	-0.4	0.5	1.4
2060	0.1	0.4	-0.6	-0.6	1.6	2.7
<i>Interest rate</i>						
2020	0.0	0.0	0.0	0.0	0.0	-0.1
2040	0.0	0.0	0.1	0.1	0.0	-0.2
2060	0.0	-0.1	0.1	0.1	-0.2	-0.4
<i>Consumption tax rate</i>						
2020	-0.1	-0.1	0.7	0.2	-4.3	-1.1
2040	-0.1	-0.1	1.1	0.4	-4.3	-1.1
2060	-0.2	-0.2	1.2	0.5	-4.3	-1.1
<i>Contribution rate</i>						
2020	-0.5	-0.3	0.2	0.1	-6.0	-10.6
2040	-0.7	-0.4	0.8	0.6	-6.0	-10.6
2060	-0.6	-0.5	0.8	0.9	-6.0	-10.6
<i>Mean retirement age (in month)</i>						
2020	0.0	1.2	3.6	0.0	1.2	0.0
2040	12.0	2.4	3.6	0.0	3.6	3.6
2060	15.6	2.4	3.6	0.0	8.4	8.4

Table 3-6: Welfare and efficiency effects of the retirement age increase

Birth year	Age in 2009	<i>NRA 67</i>				<i>Adjustment factor 6%</i>				
		without LSRA			with LSRA	without LSRA			with LSRA	
		by status				by status				
		<i>regular</i>	<i>disab.</i>			<i>regular</i>	<i>disab.</i>			
<i>Retirees</i>										
1920	89	-0.03	-0.03	0.00		0.08	0.08	0.00		
1940	69	0.19	0.19	0.00		0.15	0.16	0.00		
<i>Workers</i>										
		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>	<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>	
1960	49	-0.16	-0.43	-0.33	-1.11	0.00	-0.73	-0.16	-0.11	0.17
1980	29	0.39	0.03	-0.10	-	0.00	-0.32	-0.30	-0.20	-
<i>Future Gen.</i>										
2000	9		0.36		0.07		-0.08		-0.33	
2020	-		0.30		0.07		0.01		-0.33	
2040	-		0.24		0.07		0.05		-0.33	
2060	-		0.19		0.07		0.07		-0.33	

In percent of remaining resources.

As one can see in both cases in which we try to raise the retirement age, retirees benefit from reduced consumption taxes. Especially elderly workers are hurt since they do not benefit from reduced contribution rates while they have to accept a reduction in benefits.¹³ Younger and future cohorts experience welfare gains since they now have to pay lower pension contributions. The general redistribution pattern seems to be quite similar in both simulations. Note, however that the increase in the adjustment factor hurts especially low productivity households while at the same time future cohorts benefit considerably less than in the first simulation. Although the welfare results are so similar, the efficiency calculations show a remarkable difference. Whereas the increase in the normal retirement age comes along with a slight efficiency gain of 0.07 percent of remaining aggregate resources, the higher adjustment factor reduces aggregate efficiency significantly by 0.33 percent.¹⁴ This difference can be explained as follows. When the normal retirement age is increased, disabled individuals are hardly affected by this reform. Consequently, the efficiency gain reflects mainly reduced labor market distortions. On the other

¹³Note that our model does not allow those already retired to re-enter the work force. Although there are hardly any institutional barriers for working beyond normal retirement age, our approach reflects the actual pattern in Germany quite well.

¹⁴Note that aggregate resources include leisure. Welfare figures would more than double, if we would only account for remaining goods consumption.

hand, when the adjustment factor is increased, then disabled workers experience a massive benefit reduction since their maximum adjustment (for retirement before age 60) increases from 10.8 to 18 percent. Since disability risk is higher for low productivity households, they experience much higher welfare losses compared to the previous simulations. Of course, this also explains the significant efficiency loss since the insurance properties of the system are strongly reduced.

Table 3-7: Welfare and efficiency effects of higher progressivity

Birth year	Age in 2009	<i>Correction factor for LE</i>				<i>EP upgrade</i>					
		without LSRA			with LSRA	without LSRA			with LSRA		
		by status				by status					
<i>Retirees</i>		<i>regular</i>		<i>disab.</i>		<i>regular</i>		<i>disab.</i>			
1920	89	-0.81		-0.81	0.00	-0.29		-0.29	0.00		
1940	69	-0.93		-0.98	0.00	-0.39		-0.41	0.00		
<i>Workers</i>		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>	<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>		
1960	49	4.98	0.38	-1.06	-1.18	0.00	1.08	0.07	-0.10	-0.08	0.00
1980	29	2.77	0.27	-0.84	-	0.00	1.96	0.03	0.05	-	0.00
<i>Future Gen.</i>											
2000	9			0.15		0.30			0.27		0.13
2020	-			-0.11		0.30			0.01		0.13
2040	-			-0.15		0.30			-0.16		0.13
2060	-			-0.16		0.30			-0.19		0.13

In percent of remaining resources.

Table 3-7 reports the welfare results for the two reforms where we increase the progressivity of the German pension system. In both cases the consumption tax rate increases which explains welfare losses for already retired households. Not surprisingly especially low productivity working age cohorts benefit from the reform. In the left part of Table 3-7 there is a clear redistribution from high towards low productivity individuals, in the right part redistribution is less pronounced. Due to rising consumption taxes and contribution rates, young and future cohorts are hurt by both reforms. Of course, higher progressivity increases distortions in the labor market. But it also improves the insurance properties of the pension system. Since the latter dominates the former, the overall efficiency effect is positive in both cases considered.

Table 3-8 reports the welfare results from the privatization experiment. Retirees experience

Table 3-8: Welfare and efficiency effects of pension privatization

Birth year	Age in 2009	<i>Payroll taxation</i>				<i>Payroll/consumption taxation</i>				
		without LSRA			with LSRA	without LSRA				with LSRA
		by status				by status				
		<i>regular</i>	<i>disab.</i>			<i>regular</i>	<i>disab.</i>			
<i>Retirees</i>										
1920	89	1.88	1.93	0.00		0.47	0.48	0.00		
1940	69	1.77	1.87	0.00		0.41	0.43	0.00		
<i>Workers</i>										
		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>
1960	49	2.41	2.05	1.72	2.00	0.00	1.55	1.41	1.08	0.39
1980	29	-2.18	-2.98	-2.63	-	0.00	-1.79	-2.85	-2.86	-
<i>Future Gen.</i>										
2000	9		0.07		1.13		0.64		0.98	
2020	-		2.05		1.13		2.69		0.98	
2040	-		3.82		1.13		4.43		0.98	
2060	-		4.15		1.13		4.74		0.98	

In percent of remaining resources.

welfare gains due to the cut in consumption taxes. Obviously, these gains are much lower in the case of partially consumption tax financed pension claims. For working cohorts the effect is twofold. Older workers, on the one hand, only face modest cuts in pension benefits and enjoy significant decreases in payroll and consumption taxes which increases welfare. Note that this increase is lowest for high income earners, since their income tends to be above the contribution ceiling. Younger workers, on the other hand, face serious welfare losses due to stronger pension benefit cuts. Finally, the reduction in payroll taxes increases disposable income at younger ages and therefore loosens liquidity constraints. Therefore the generation born in year 2000 experiences slight welfare gains. Furthermore, as private savings and thereby accidental bequests successively increase, the positive liquidity effect is enforced. Hence, welfare continuously rises throughout the transition. The compensated welfare changes indicate that the elimination of social security induces an aggregate efficiency gain between 0.98 and 1.13 percent of aggregate resources. The reported aggregate efficiency effects of the privatization experiments in Table 3-8 are due to (i) the elimination of longevity insurance through pension payments, (ii) the increase in income taxation and reduction in consumption taxes and (iii) the relaxation of liquidity constraints. Since pension contributions were exempt from tax, the re-

duction in the payroll tax rate causes an increase in the tax base during high productive years. Furthermore, the rise in private assets broadens the tax base of capital income taxation. In models with deterministic income the switch towards income taxation reduces economic efficiency as already shown in Auerbach and Kotlikoff (1987). With stochastic income, the explanation is complicated by the implied insurance effects of the tax structure. As Nishiyama and Smetters (2005) have recently shown, consumption tax systems provide less insurance against income shocks than progressive income tax systems and this might even outweigh the increased labor supply distortions. Due to this reasoning, the efficiency effect of our reform is lower in the case of partially consumption taxed financed pension claims. Overall, we find that the positive liquidity effects of the pension reform more than offset the loss in longevity insurance. Hence, efficiency is positive in both simulations.

3.4 Sensitivity analysis with respect to the model structure

In this section we present some sensitivity analysis with respect to the preference structure and the income uncertainty. In order to isolate risk aversion from intertemporal substitution, we follow the approach of Epstein and Zin (1991) and rewrite the preference structure of the representative consumer. Table 3-9 reports the welfare results when we assume that households are completely risk averse.

Table 3-9: Sensitivity analysis: Higher progressivity with risk neutral individuals

		<i>Correction factor for LE</i>					<i>EP upgrade</i>				
		<i>regular</i>		<i>disab.</i>			<i>regular</i>		<i>disab.</i>		
<i>Retirees</i>											
1920	89	-0.74		-0.74	0.00		-0.25		-0.25	0.00	
1940	69	-0.85		-0.89	0.00		-0.34		-0.35	0.00	
<i>Workers</i>		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>	
1960	49	4.86	0.40	-0.97	-1.16	0.00	1.03	0.09	-0.09	-0.48	0.00
1980	29	1.89	0.11	-0.86	–	0.00	1.05	-0.02	0.02	–	0.00
<i>Future Gen.</i>											
2000	9		-0.25		-0.52			-0.07		-0.22	
2020	–		-0.45		-0.52			-0.30		-0.22	
2040	–		-0.49		-0.52			-0.43		-0.22	
2060	–		-0.49		-0.52			-0.46		-0.22	

In percent of remaining resources.

Note that the reported welfare effects have the same pattern as in Table 3-7. The main difference is that young and future cohorts experience significantly higher welfare losses. Of course this is due to the fact that risk neutral households do not need an insurance. They do not benefit from the improved insurance provision. Consequently, aggregate efficiency effects now turn negative. These numbers reflect the increased labor supply distortions induced by higher progressivity. The difference between the efficiency numbers in Tables 3-9 and 3-7 quantify the insurance effect which risk averse consumers experience. Of course, the insurance gains increase with risk aversion.

Finally, in Table 3-10 we assume that households face no income uncertainty and no disability risk. Since we only consider a representative household per cohort, we can't distinguish between different productivity types within a cohort anymore. The structure of intergenerational welfare effects is very similar as in the previous simulation with uncertain income. Pensioners gain, younger working cohorts lose and future generations gain again. However, now privatization induces efficiency losses in the first simulation. This is due to two facts. On the one hand, the increase in income taxes now only distorts labor supply but does not provide income insurance. On the other hand, with certain income household save much more at the beginning of the life cycle and liquidity constraints are basically absent. Therefore, the loss in longevity insurance dominates any other effects and aggregate efficiency decreases. In the case of partially consumption tax financed pension claims, however, the switch towards a consumption tax regime now induced a rise in efficiency as explained in Auerbach and Kotlikoff (1987) and therefore the reform comes along with a slight efficiency gain.

4 The multi-regional world model

We now turn to our multi-regional world model which is based on previous work in Fehr et al. (2007, 2008, 2010). The model covers five different world regions, namely the U.S., the European Monetary Union (EMU), Northeast Asia (NEA) - consisting of Japan, Korea, Taiwan, and Hong Kong -, China, and India. In each region there coexist 90 overlapping generations with each cohort split into three skill classes. Production distinguishes six goods – two traded and two non-traded consumer goods, a traded investment good and a non-traded public con-

Table 3-10: Sensitivity analysis: Pension privatization with certain income

		<i>Payroll taxation</i>				<i>Payroll/consumption taxation</i>					
<i>Retirees</i>		<i>regular</i>		<i>disab.</i>		<i>regular</i>		<i>disab.</i>			
1920	89	2.62		–	0.00	1.54		–	0.00		
1940	69	2.51		–	0.00	1.47		–	0.00		
<i>Workers</i>		<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>	<i>low</i>	<i>medium</i>	<i>high</i>	<i>disab.</i>		
1960	49	–	2.89	–	–	0.00	–	2.42	–	–	0.00
1980	29	–	-4.40	–	–	0.00	–	-4.18	–	–	0.00
<i>Future Gen.</i>											
2000	9	-1.58		-0.12		-1.17		0.17			
2020	–	0.15		-0.12		0.60		0.17			
2040	–	1.71		-0.12		2.16		0.17			
2060	–	2.03		-0.12		2.46		0.17			

In percent of remaining resources.

sumption good. In each sector the production technology includes heterogeneous labor inputs, so that each skill level earns a wage that can vary relative to those of the other skill groups. The model also features age-specific demands for consumption goods and services (e.g., health-care). Capital is assumed to be internationally mobile while labor is immobile besides the exogenously specified immigration. International and sectoral factor price equalization holds.

The following sections describe the model structure in more detail.

4.1 General model structure

4.1.1 Demographics

Agents in each region live at most to age 90. Consequently, there are 91 generations with surviving members at any point in time, i.e. each period corresponds to one year. The life cycle of a representative agent is described in Figure 4-1. Between ages 0 and 20 our agents are non-working children supported by their parents. At 21 our agents go to work and become individual households. Between ages 23 and 45 our agents give birth each year to fractions of children. An agent's first-born children (fractions of children) leave home when the parents are age 43 and the last-born leave when the agents are age 66. Our agents die between ages 68 and 90. The probability of death is 1 at age 91. Children always outlive their parents, meaning that

parents always outlive grandparents.¹⁵

FIGURE 4-1 HERE.

We denote the population vector for year t by $N(a, t, k)$, where $a = 1, \dots, 90$ denotes the agent's age and $k = 1, 2, 3$ is the skill class. Since agents younger than 23 have no children and those over 65 have only adult children, the total number of children of an agent age a with skill k in year t $KID(a, t, k) = 0$ for $0 \leq a \leq 22$ and $66 \leq a \leq 90$. Agents between these ages have economically dependent children. Take, for example, a 30 year-old agent. Such an agent has children who were born in the years since she/he was 23. In year t , these children are between age 0 and 7. We calculate the number of children in each household aged a , $KID(\cdot)$, as the sum of the total number of children of the respective parent skill-class generation divided by the total number of parents of age a in year t who belong to skill k . This function takes into account that the family's age structure will change over time due to changes over time in age-specific fertility.

Our treatment of immigration is, by necessity, simple. In each year new immigrants in each skill and age group arrive with the same number and age distribution of children and the same level of assets as natives of the identical skill and age. Once they join a native cohort, they experience the same future age-specific fertility and mortality rates as native-born agents.

4.1.2 The household sector

The model's preference structure is represented by a time-separable, nested, CES utility function. Remaining lifetime utility $U(l, t, k)$ of a generation age l at time t who belong to skill-class k takes the form

$$U(l, t, k) = V(l, t, k) + H(l, t, k), \tag{4-1}$$

where $V(l, t, k)$ records the agent's utility from her/his own goods and leisure consumption and $H(l, t, k)$ denotes the agent's utility from the consumption of her/his children. The two

¹⁵If a parent reaches age 90, his or her oldest children will be 67. These are children who were born when the parent was age 23.

sub-utility functions are defined as follows:

$$V(l, t, k) = \frac{1}{1 - \frac{1}{\gamma}} \sum_{a=l}^{90} \left(\frac{1}{1 + \delta} \right)^{a-l} P(a, i) \left[\bar{c}(a, i, k)^{1-\frac{1}{\rho}} + \varepsilon \ell(a, i, k)^{1-\frac{1}{\rho}} \right]^{\frac{1-\frac{1}{\gamma}}{1-\frac{1}{\rho}}} \quad (4-2)$$

$$H(l, t, k) = \frac{1}{1 - \frac{1}{\gamma}} \sum_{a=l}^{90} \left(\frac{1}{1 + \delta} \right)^{a-l} P(a, i) KID(a, i, k) \bar{c}_K(a, i, k)^{1-\frac{1}{\gamma}}, \quad (4-3)$$

where

$$\bar{c}(a, i, k) = \left[\sum_{j \in G_c} \kappa(j, a) c(j, a, i, k)^\varphi \right]^{\frac{1}{\varphi}} \quad (4-4)$$

denotes the aggregate private consumption good, while $c(j, a, i, k)$ and $\ell(a, i, k)$ denote private consumption of goods j and leisure respectively, G_c defines the set of all private consumption goods, $i = t + a - l$ and $\kappa(j, a)$ defines the consumption share of good j at age a , with $\sum_{j \in G_c} \kappa(j, a) = 1$. In permitting these shares to vary with age, our model can accommodate age-related changes in, for example, demand for medical services. The elasticity of substitution between different consumption goods j is denoted by ω , where $\varphi = 1 - \frac{1}{\omega}$. The price index $\bar{p}(a, i)$ for the aggregate consumption good $\bar{c}(a, i, k)$ at age a in year i is

$$\bar{p}(a, i) = \left[\sum_{j \in G_c} \kappa(j, a)^\omega p(j, i)^{1-\omega} \right]^{\frac{1}{1-\omega}}, \quad (4-5)$$

where $p(j, i)$ defines the consumer price of good j in year i .¹⁶ The aggregate consumption of children of skill-class k parents who are age a in period i is defined as $\bar{c}_K(a, i, k)$, and the number of children supported by parents age a is $KID(a, i, k)$. We assume that parents apply the same consumption shares $\kappa(j, a)$ and substitution elasticities ω when they decide upon their children's consumption $c_K(j, a, i, k)$.

Since lifespan is uncertain, the utility of consumption in future periods is weighted by the survival probability of reaching age a in year i

$$P(a, i) = \prod_{u=l}^a [1 - d(u, u - a + i)], \quad (4-6)$$

¹⁶Note that the price index is independent of skill class since we assume identical consumption shares in each skill class.

which is determined by multiplying the conditional survival probabilities from year t (when the agent's age is l) through year i . Note that $d(l, t)$ is the mortality probability of an agent age l in year t . The parameters $\delta, \rho, \varepsilon$ and γ represent the rate of time preference, the intratemporal elasticity of substitution between consumption and leisure, the leisure preference parameter, and the intertemporal elasticity of substitution between consumption and leisure, respectively.

Given the asset endowment $a(l, t, k)$ of the agent in year t , the flow budget constraint is defined by:

$$a(l+1, t+1, k) = [a(l, t, k) + I(l, t, k)](1 + r(t)) + w(t, k)E(l, t)[h(l, t) - \ell(l, t, k)] - T(l, t, k) - \bar{p}(l, t)[\bar{c}(l, t, k) + KID(l, t, k)\bar{c}_K(l, t, k)], \quad (4-7)$$

where $r(t)$ is the pre-tax return, $I(l, t, k)$ denotes inheritances received in year t . The budget incorporates our assumption that annuity markets are not operative. Instead, agents who die leave unintended bequests. Private assets of all agents who died are aggregated and then distributed according to an exogenous age-dependent distribution scheme $\Gamma(l)$ to all agents aged between 21 and 49 which follows a normal distribution. To be precise, the inheritance of agents age l in year t is given by:

$$I(l, t, k) = \Gamma(l) \frac{\bar{A}(t, k)}{N(l, t, k)} \quad \text{where} \quad \sum_{l=21}^{49} \Gamma(l) = 1. \quad (4-8)$$

The numerator in this ratio measures the aggregate assets of skill-class k agents who die in year t . A share $\Gamma(l)$ of these bequests is dedicated to inheritants aged l of the same skill class. This share is split equally among all agents of the same age group.

As in Altig et al. (2001) and Kotlikoff et al. (2007), we model technical progress as permitting successive generations to use time more effectively, whether in working or enjoying leisure. We implement this assumption by having the time endowment of successive generations in each region grow at the common rate λ . Denote $h(a, i)$ as the time endowment of an agent age a at time i , then

$$h(a, i) = (1 + \lambda)h(a, i - 1). \quad (4-9)$$

This treatment of technical change ensures eventual convergence of the economy to a long-run steady state. Other formulations of technical change, such as making it labor-augmenting, preclude a steady state given the model's preferences. And our iterative method for determining the model's equilibrium transition path requires the terminal conditions provided by the economy's long-run steady state.¹⁷

Gross labor income of an agent in year i is derived as the product of her labor supply and wage rate. The latter is the product of the skill-specific wage rate $w(i, k)$ in year i and age- and year-specific productivity per time-unit $E(a, i)$.

Net taxes, $T(l, t, k)$, include consumption, capital income, and progressive wage taxes as well as social security contributions net of pension, disability, and health benefits received in the form of transfer payments.

Given price indices $\bar{p}(a, i)$, interest rates $r(i)$, and wages $w(i, k)$, agents maximize utility (4-1) subject to the intertemporal budget constraint (4-7) and the constraint that leisure in each period not exceed the time endowment (i.e. $\ell(l, t, k) \leq h(l, t)$). They do this by choosing their leisure and consumption demands, i.e., $\ell(a, i, k)$, $\bar{c}(a, i, k)$ and $\bar{c}_K(a, i, k)$. Demands for specific goods $j \in G_c$ are, then, derived from

$$c(j, a, i, k) = \left(\frac{\kappa(j, a)}{p(j, i)} \right)^\omega \bar{p}(a, i)^\omega \bar{c}(a, i, k). \quad (4-10)$$

Given individual consumption and leisure, agents' asset levels are derived from (4-7). Aggre-

¹⁷Note that assuming a higher rate of technical progress is isomorphic to assuming a higher rate of fertility; i.e., having more people is equivalent to having fewer people who each have more time. Since fertility rates don't enter into production functions, we circumvent the problem of steady-state incompatibility.

gate values of assets, bequests, private consumption goods, and labor supply obey

$$A(t+1) = \sum_{k=1}^3 \sum_{a=21}^{90} a(a+1, t+1, k)N(a, t, k), \quad (4-11)$$

$$\bar{A}(t+1, k) = \sum_{a=21}^{90} d(a+1, t+1)a(a+1, t+1, k)N(a, t, k), \quad (4-12)$$

$$C(j, t) = \sum_{k=1}^3 \sum_{a=21}^{90} [c(j, a, t, k) + KID(a, t, k)c_K(j, a, t, k)]N(a, t, k), \quad (4-13)$$

$$L^s(t, k) = \sum_{a=21}^{90} E(a, t)[h(a, t) - \ell(a, t, k)]N(a, t, k). \quad (4-14)$$

Since households die at the beginning of each period, we aggregate assets across all agents alive at the end of the prior period and multiply with the probability of dying in the respective period to compute $\bar{A}(t+1, k)$, which is used in the calculation of inheritances (see (4-8)). Total assets of agents alive at the end of period $t+1$ satisfies

$$\mathcal{A}(t+1) = \sum_{k=1}^3 \sum_{a=21}^{90} a(a, t+1, k)N(a, t+1, k), \quad (4-15)$$

which includes the assets and numbers of period $t+1$ immigrants.

4.1.3 The production sector

Besides private consumption goods, each region also produces a public consumption and an investment good, the index set of all goods produced is denoted by G . Aggregate output $Y(j, t)$ of each good j is produced via a Cobb-Douglas technology that uses capital $K(j, t)$ and skill-specific labor $L(j, t, k)$, i.e.,

$$Y(j, t) = \phi K(j, t)^{\alpha(j)} \left[\prod_{k=1}^3 L(j, t, k)^{\beta(j, k)} \right]^{1-\alpha(j)}, \quad \text{for } j \in G \quad (4-16)$$

where $\sum_{k=1}^3 \beta(j, k) = 1$. The parameter ϕ references total factor productivity, and $\alpha(j)$ and $\beta(j, k)$ denote capital's share and the share of skill-specific labor inputs in production, respectively.

Profit maximization requires

$$[r(t) + \delta_K]K(j, t) = \alpha(j)q(j, t)Y(j, t) \quad (4-17)$$

$$w(t, k)L(j, t, k) = [1 - \alpha(j)]\beta(j, k)q(j, t)Y(j, t), \quad (4-18)$$

where δ_K is the depreciation rate and $q(j, t)$ denotes the producer price of good j in t .

4.1.4 The government sector

Each region's government issues additional debt, $\Delta B(t)$, and collects taxes to finance outlays for general government expenditures $q(g, t)C(g, t)$ (with $q(g, t)$ and $C(g, t)$ defining producer price and quantity, respectively), general-revenue financed social benefits $SB(t)$ (i.e. pension, healthcare, and disability benefits) and interest on existing debt:

$$\Delta B(t) + \sum_{k=1}^3 \sum_{a=21}^{90} T(a, t, k)N(a, t, k) = q(g, t)C(g, t) + \varrho SB(t) + r(t)B(t), \quad (4-19)$$

where ϱ denotes the share of these transfer payments that are financed by general revenues.

Each government maintains its initial debt-to-output ratio over time. The progressivity of wage taxation is modelled after Auerbach and Kotlikoff (1987), with marginal wage tax rates rising linearly with the wage-tax base. $PY(t)$ defines the aggregate payroll-tax base, which differs from total labor earnings due to the ceiling on taxable wages. This ceiling is fixed at 290, 200, 150, 300, and 300 percent of average income in the U.S., EMU, NEA, China, and India, respectively. Average employer plus employee payroll tax rates $\hat{\tau}^p(t)$ for the pension, healthcare, and disability transfer programs are determined based on each region's transfer-program-specific budget, taking into account general revenue finance, i.e.

$$\hat{\tau}^p(t)PY(t) = (1 - \varrho)SB(t). \quad (4-20)$$

Due to contribution ceilings, individual payroll-tax rates can differ from the average payroll tax rate. Above the contribution ceiling, marginal social security contributions are zero and average social security contributions fall with the agent's income. To accommodate this non-convexity

in the budget constraint, we assume that the highest earnings class in each region pays payroll taxes up to the relevant ceilings, but faces no payroll taxation at the margin.

If an agent retired in year i at the exogenously set retirement age $\bar{a}(i)$, her pension benefit $Pen(a, t, k)$ in year $t \geq i$ when she is age $a \geq \bar{a}(i)$ is assumed to depend linearly on her average earnings during her working life $\bar{W}(i, k)$:

$$Pen(a, t, k) = \nu_0 + \nu_1 \times \bar{W}(i, k). \quad (4-21)$$

The region-specific parameters ν_0, ν_1 for the U.S., the EMU and NEA were chosen to match replacement rates reported in OECD (2009). In China and India, only a fraction of public employees are covered by the public pension system. But spending on civil service pensions in China and India amounts to approximately 2 percent of the respective GDP (see ILO 2010). Since we do not distinguish between covered and non-covered employment in our model, we assume a pension-replacement rate of 30 percent of average pre-retirement earnings. The rate is far too low for covered employees and far too high for non-covered employees, but it results in realistic aggregate pension expenditures in the two countries.

General government expenditures $q(g, t)C(g, t)$ consist of government purchases of goods and services, including educational expenditures and health outlays. Over the transition, we keep age-specific per capita purchases fixed in efficiency terms, so that aggregate expenditures adjust due to population dynamics.¹⁸

Age-specific health outlays per capita also grow with GDP per capita. However, in the U.S., EMU, and Japan we assume an additional growth rate of 2.0 percent per year until 2025 and of 1.0 percent between 2025 and 2035.¹⁹ In China and India, age-specific healthcare outlays per capita are assumed to grow at a faster pace: During the first 40 years of the transition there is an additional annual growth rate of four percent. Note that while we treat 80 percent of health benefits as government consumption and 20 percent as fungible transfers to households,

¹⁸Implicitly, we assume that additive separability of preferences in public consumption goods. But since the latter is constant during the transition, we can omit it in (4-1).

¹⁹As shown in Hagist and Kotlikoff (2009), this is a rather conservative assumption concerning future growth in benefit levels. In the U.S., for example, the 1970-2002 average annual growth rate of real government healthcare benefit exceeded the average annual growth rate of per capita GDP by 2.6 percentage points.

disability benefits are modeled exclusively as fungible transfers to households.

During the transition, we keep the ratio of wage-tax to consumption-tax revenue fixed each year and balance the government's annual budget (4-19) by adjusting the intercept in our linear equation determining the average wage-tax rate as well as the consumption-tax rate.

4.1.5 Computational approach for solving the dynamic world equilibrium

To compute the world economy's perfect-foresight general equilibrium transition path, we normalize the producer price of the investment good to unity and start with initial guesses of the equilibrium factor prices and product prices of consumption goods. Given these prices we can determine consumption demands as well as labor and asset supplies for each region. Summing up demand of traded goods and the difference between future and current assets (net of public debt) across regions determines the required world output for traded goods and the investment good. Next we determine the factor demands in the non-traded goods sectors and derive the remaining factor supplies for traded goods in each region. Using the fact that world factor supplies have to be equal to world factor demands for traded goods, we can update factor prices and producer prices and compute region-specific shares of production for each traded good. Our algorithm iterates until the time paths of interest and wage rates converge to a fixed point and supplies for each good equals its demand. We give our economy 300 years to reach to a steady state. In fact, our model reaches a steady state to many decimal places decades prior to year 300. It also converges very tightly around the equilibrium transition path.

However, nothing precludes our calculating negative values for the region-specific output shares of traded goods. Negative values are, of course, infeasible and indicative of specialization. To deal with specialization, we assume that domestic firms employ offshore labor to participate in their domestic production processes via internet and telephone until the negative output shares have disappeared. Of course, this offshoring scenario restores factor price equalization across regions despite the fact that some goods are not produced in a country.

In the appendix we explain in detail how this mechanism is implemented numerically.

4.2 Calibration of the baseline path for the world economy

4.2.1 Population projections

This section describes our data set for the benchmark population in the year 2000 and the transition.²⁰ The main data source for our population data is the medium variant of the United Nations population projections (UNPD 2007). In the case of Taiwan we had to rely on national population statistics provided by the Statistical Office (CEPD 2007). However, part of the raw data was only supplied in aggregates. In addition, the specific structure of our population model imposed certain restrictions on our data set.

To specify the current and future demographic structure of each region we start with year-2000 age-specific population and age-specific net-immigration counts. Each cohort in our model is split into three skill classes k . We assume that 15 percent of each cohort belong to the lowest skill class, 30 percent to the top skill class, and the remaining 55 percent to the middle skill class in the U.S., EMU and NEA. This is in line with figures on educational attainment from Barro and Lee (2001) for the three-region agglomerate as well as with the latest figures for the U.S. from the U.S. Census Bureau (2008). In China and India, we assume that 22 percent of the overall population belong to the lowest, 25 percent to the top skill class, and 53 percent to the middle. At first glance, the small value for the share of low-skilled workers in China and India may seem surprising. Indeed, according to the data of Barro and Lee (2001), 44 percent of the population aged 15 and over in India and 18 percent in China have received no formal education. On the other hand, these shares produce relative wages for the three income classes in the initial year that coincide with OECD (2008) region-specific earnings-dispersion data.

Given the population age structure in year 2000 as well as projected future fertility, mortality, and net immigration rates, we compute the population vector $N(a, t, k)$ for the years t between 2001 and 2050. After year 2050, fertility rates are endogenously adjusted in line with zero population growth and a stable population age structure.

Table 4-1 compares the actual and simulated changes over time in fertility rates, life expectan-

²⁰Although the economic model starts in year 2008, we chose year 2000 as the initial year for the population projections due to data availability.

Table 4-1: Comparing actual and simulated population projections

Country Year	U.S.		EMU		NEA		China		India	
	2008*	2050	2008*	2050	2008*	2050	2008*	2050	2008*	2050
<i>Fertility Rate</i>										
Model	2.06	1.85	1.51	1.82	1.55	1.75	1.66	1.85	2.80	1.85
Official	2.05	1.85	1.50	1.85	1.53	1.73	1.73	1.85	2.81	1.85
<i>Life Expectancy at Birth</i>										
Model	82.0	83.8	82.6	84.6	84.5	87.3	76.5	80.2	64.6	75.3
Official	78.2	83.1	79.7	84.3	81.2	86.7	73.0	79.3	64.7	75.6
<i>Total Population (in Millions)</i>										
Model	303.6	403.6	312.0	308.3	206.9	181.4	1314.2	1401.5	1184.9	1643.9
Official	299.8	402.4	312.2	312.9	205.6	172.6	1313.0	1408.8	1134.4	1658.3
<i>Age Structure (in Percent of total Population)</i>										
<i>0-15 Years</i>										
Model	20.3	17.8	15.6	15.0	14.7	13.7	20.2	16.3	32.0	18.2
Official	20.8	17.3	15.6	14.5	15.5	10.7	21.6	15.3	33.0	18.2
<i>15-64 Years</i>										
Model	67.4	62.0	67.0	57.5	64.2	53.1	72.9	61.6	63.4	66.1
Official	66.9	61.7	66.8	55.9	68.5	52.6	70.7	61.0	62.0	67.3
<i>65-90 Years</i>										
Model	12.4	20.1	17.4	27.5	21.2	33.2	6.9	22.0	4.6	15.7
Official	12.3	21.0	17.6	29.6	16.0	36.7	7.7	23.7	5.0	14.5

* year 2005 in the official population data

cies, total populations and population age structures. Due to relatively high fertility and net immigration, the U.S. population is projected to increase from 304 million in 2008 to 404 million in 2050. In the EMU, the population falls until mid-century from 312 to 308 million. In Northeast Asia, the population falls from 207 million to 181 million. In contrast, the Chinese population increases from roughly 1.31 billion to 1.40 billion, and the population in India increases from 1.18 billion in 2008 to 1.64 billion in 2050; i.e., India becomes the most populous country in the world.

As one would expect, the population share of those 65 and older increases in all five regions. There are, however, major differences in the aging process across the regions. First, whereas the share of the working-age population increases in India until 2050, it decreases modestly in the U.S., but more substantially in the EMU, China and NEA. Second, the share of elderly increases to a much larger extent in NEA and China compared to India, the EMU, and the U.S.

4.2.2 Production, preference and policy parameters

Our aggregation of goods into four consumption goods (services, housing, low tech, and high tech), an investment good, and a public good is based on the March 2007 release of the EU-KLEMS database²¹. Given this sectoral aggregation, we computed the capital income shares as well as the skill-specific labor shares from the 2004 (SIC based) U.S. data as reported in Table 4-2. As one would expect, capital income shares are especially high in the housing sector; they are especially low in the private service and public goods sectors. Note that with the exception of housing our capital shares accord closely with the sector capital shares reported in Valentinyi and Herrendorf (2008). As indicated in the table, the low-tech consumption good and housing sectors have the highest shares of low-skilled labor; the high-tech consumption good sector, the non-housing services sector, and the public good sector have the highest share of high-skilled labor. Finally, the depreciation rate is set at the same rate for all countries.

Table 4-2: Production Technology Parameters

	Symbol	Services	Housing	Low tech	High tech	Investment Good	Public Good
Capital share in production	$\alpha(j)$	0.26	0.57	0.44	0.41	0.35	0.26
Share of specific labor inputs	$\beta(k, j)$						
Low-skill ($k = 1$)		0.08	0.11	0.15	0.03	0.06	0.02
Medium-skill ($k = 2$)		0.57	0.62	0.58	0.38	0.57	0.39
High-skill ($k = 3$)		0.35	0.27	0.27	0.59	0.37	0.59
Technology coefficient	ϕ				4.25*		
Depreciation rate	δ_K				0.075		

* Normalizes U.S. low-skilled wages in initial year to unity.

Table 4-3 reports values of preference and policy parameters. The time-preference rates in the five regions were set to match the model's 2008 region-specific ratios of private consumption to GDP in the developed regions. Given this time preference rate, we increased general public expenditures in order to reduce private consumption in China and India. The intertemporal elasticity of substitution, the elasticity of substitution between consumption and leisure, and the leisure preference parameters are taken from Kotlikoff et al. (2007). The elasticity of sub-

²¹See Timmer et al. (2007). Fehr et al. (2008) provide a detailed description of the sectoral mapping.

stitution between consumption goods was chosen to generate a pattern of demand that accords with U.S. consumption data. The age-specific weights of the different consumption goods in the utility function are derived from U.S. Department of Labor (2007), where we aggregated the different consumption goods reported there according to our classification. Note that the consumption shares vary significantly with age, with housing's share rising considerably with age.

Table 4-3: Preference, Productivity and Policy Parameters

	Symbol	U.S.	EMU	NEA	China	India
Time preference rate	δ			0.01		
Intertemporal elasticity of substitution	γ			0.25		
Intratemporal elasticity of substitution						
-between consumption and leisure	ρ			0.4		
-between consumption goods	ω			1.1		
Leisure preference parameter	ε			1.5		
Consumption shares of goods j	$\kappa(j, a)$	Services	Housing	Low tech	High tech	
$a < 25$		0.04	0.36	0.19	0.41	
$25 \leq a \leq 34$		0.06	0.40	0.17	0.37	
$35 \leq a \leq 44$		0.07	0.40	0.18	0.35	
$45 \leq a \leq 54$		0.08	0.38	0.17	0.37	
$55 \leq a \leq 64$		0.11	0.38	0.17	0.34	
$65 \leq a \leq 74$		0.16	0.38	0.16	0.30	
$a \geq 75$		0.21	0.40	0.15	0.24	
Shift parameter for productivity	ξ	1.00	0.60	0.45	0.053	0.028
Technical progress	λ			0.01		
Capital tax rate (in %)	τ^r	11.0	14.0	8.0	3.0	3.0
Debt (in % of national income)	B/Y	70.0	76.0	146.0	21.0	72.0
Age of retirement	\bar{a}	63	60	60	60	60

The age- and year-specific productivity profile of an age a individual in period i

$$E(a, i) = \xi(i)e^{4.47+0.033(a-20)-0.00057(a-20)^2}(1 + \lambda)^{a-21},$$

is taken – apart from the term $\xi(i)$ – from Auerbach and Kotlikoff (1987). Note that the higher is the rate of technological change, λ , the steeper is the age-ability profile. This captures the role of technical progress in influencing not just the level, but also the shape of longitudinal age-earnings profiles. The labor productivity parameter ξ is normalized at 1 for the U.S. The initial

values of this parameter for the other four regions are set to match the 2008 relative values of GDP and are gradually raised to 1 for each successive cohort of new worker cohorts over time. For the EMU and NEA we assume this adjustments occurs over the 10 years between 2009 and 2018. For China we assume it takes 15 years. Since during the last four decades India made little progress in increasing even elementary educational attainment, see Bosworth and Collins (2008), this period in India is set to 75 years. Once the phase-in period is complete, it takes another 40 years until all cohorts of workers have the same labor productivity.

The model's debt-to-GDP levels were set according to total government debt reported by the World Bank (2010). During the transition debt-to-GDP ratios are assumed to remain unchanged. The maximum ages of retirement are taken from OECD (2006a) for the U.S. and EMU and from SSA (2010) for NEA, China, and India. We calibrated the endogenous consumption and wage tax rates and set the personal capital income tax rates in order to yield the structure of indirect and direct tax revenues reported in OECD (2010b) for the U.S., the EMU, and NEA and in IMF Fiscal Affairs Department (2010) for China and India. Our wage tax systems are assumed to be progressive, with the parameters of each region's tax system set to generate what appears to be realistic average and marginal tax rates.

In calibrating health expenditures in our model, we apply the Japanese age-specific government healthcare expenditure profile for NEA as well as China and India. In the case of the EMU, we use the German profile. For the U.S. our profile comes from Hagist and Kotlikoff (2009). We assume uniform disability expenditures by age for ages between 21 and 64 in the U.S., the EMU, and NEA. We don't model separate disability programs for China and India. In the case of the U.S., EMU, and NEA, total social insurance outlays for pensions, disability, and health, measured as a share of GDP are set to accord with the values reported in OECD (2010c). In calibrating social security contribution rates, we assume that 75 percent of overall healthcare benefits in the U.S. and 25 percent in NEA are financed by general taxes. In the EMU, we assume that 20 percent of the outlays of all three social security systems (health, pension, disability) are financed by general taxes. Calibration of social insurance outlays in China and India is based on ILO (2010).

We use the German age-specific education profile (due to data availability) for all regions in the model and rescale it to get realistic education outlays in year 2008 in each region (see below). In addition to these parameter values, our model requires an initial distribution of assets by age and income class for each region. These profiles are region-specific and were adopted from a steady-state run of our simulation model.

4.2.3 Initial equilibrium and baseline path

Table 4-4 compares simulated with observed macro variables in the initial equilibrium in 2008. The official GDP figures are taken from World Bank (2010). The model's values for consumption, government purchases, and investment come very close to their official counterparts. Note however that the higher values for government expenditures in China and India in our model come from the assumed larger government sector in order to reach realistic private consumption ratios in both countries as already stated above. Our model matches the very high rate of private consumption in the U.S. and the high rates of domestic investment in China and India. We however were not able to reach such high investment rates as reported in the official data. The model does very well in matching relative GDP levels (measured in PPP). The disaggregation of public goods is based, in part, on World Bank (2010) and OECD (2010a) data on education outlays and, in part, on the aforementioned assumption that 80 percent of government health expenditures is government consumption. General public expenditures is calculated as a residual.

Our model's trade balances agree in sign with actual values, but not value – especially in the EMU und NEA. The reason is of course that our model excludes regions outside of our five, so that there is not too much concurrence with respect to trade balances and current accounts. As already explained above, outlays of the social security systems were calibrated to yield the official values from OECD (2010c) and ILO (2010). The official contribution rates for pensions, health-care and disability were taken from SSA (2009) for the U.S. and from SSA (2010) for NEA (Japan), China and India.²² Obviously, our pension and health insurance contribution rates in China and India deviate from official figures. Our model assumes that all households in

²²The official tax revenue data come from OECD (2010b) for U.S., EMU and NEA and from IMF Fiscal Affairs Department (2010) for China and India.

Table 4-4: The year 2008 of the baseline path

	Model				Official					
	U.S.	EMU	NEA	China	India	U.S.	EMU	NEA	China	India
Gross Domestic Product ^a										
Private consumption	72.2	58.7	58.8	38.5	59.9	71.2	56.4	56.0	36.8	58.1
Government purchases of goods and services	16.2	20.4	17.3	29.0	18.0	15.8	20.4	17.5	13.0	11.7
Domestic investment	20.6	17.5	16.1	25.2	26.8	18.0	22.2	25.3	42.5	35.6
Trade Balance	-9.0	3.5	7.7	7.2	-4.7	-5.1	1.0	1.2	7.7	-5.4
Current account	-0.3	2.8	6.2	-3.8	-11.9	-4.9	-0.6	2.5	9.4	-3.0
Relative GDP levels (PPP)	1.00	0.82	0.44	0.54	0.23	1.00	0.78	0.41	0.57	0.24
Government Indicators ^a										
Social benefits	14.4	19.5	15.3	4.2	3.1	14.7	19.7	15.5	4.3	3.1
Social insurance revenues	9.0	15.6	13.8	4.2	3.1					
Payroll tax rate ^b	15.3	25.9	25.2	6.9	5.3	15.3		23.6	36.0	36.1
Tax revenues ^a	20.2	23.9	25.9	28.6	20.1	19.6	24.2	17.8	16.3	12.3
Direct taxes	12.0	10.7	12.3	5.1	3.9	11.8	11.8	9.3	4.7	6.3
Wage taxes	8.2	7.1	9.8	4.8	3.3					
Capital taxes	3.9	3.6	2.4	0.3	0.6					
Indirect taxes	8.2	13.2	13.6	23.5	16.2	7.8	12.4	8.5	11.6	6.0
Wage-tax rates ^b										
Average	13.9	11.8	16.5	7.9	5.7					
Marginal	15.1	12.8	17.3	8.0	5.8					
Consumption tax rate ^b	11.3	22.5	23.1	61.0	27.0					
Capital-output ratio	2.7	2.6	2.6	2.5	2.7					

^a In percent of GDP; ^b In percent.

all regions are covered by the government's pension and health system. However, only about 20-30 percent of the people above the legal retirement age in China and India are effectively covered by public pensions and only about 6 percent of the population in India and about 24 percent of the Chinese population have formal health coverage (ILO 2010).

Table 4-5: Country-specific simulation results of the baseline path

	Year	GDP	Capital Stock	Labor Demand			Effective Wage Tax ^a	Pay-roll Tax ^a	Average Wage Tax ^a	Consumption Tax ^{a,b}
				Low	Middle	High				
U.S.	2008	1.00	1.00	1.00	1.00	1.00	39.3	15.3	13.9	11.3
	2020	1.14	0.96	1.18	1.22	1.32	48.3	21.1	16.1	12.5
	2040	1.59	1.21	1.92	1.77	1.91	58.3	26.5	19.4	14.1
	2060	2.57	2.16	3.10	2.74	2.87	60.5	23.4	20.5	19.9
EMU	2008	0.82	0.79	0.65	0.78	0.95	56.2	26.0	11.8	22.5
	2020	0.90	0.74	0.80	0.92	1.16	63.0	32.9	12.2	21.8
	2040	1.22	0.90	1.31	1.30	1.60	72.7	41.3	13.3	22.1
	2060	1.76	1.46	1.94	1.81	2.10	73.2	37.4	13.9	28.7
NEA	2008	0.44	0.42	0.34	0.41	0.50	60.4	25.3	16.5	23.1
	2020	0.48	0.39	0.44	0.49	0.60	68.9	33.3	17.4	22.3
	2040	0.67	0.49	0.78	0.76	0.85	76.9	35.9	20.2	26.2
	2060	0.95	0.78	1.16	1.03	1.06	74.1	32.6	19.3	28.5
China	2008	0.54	0.51	0.53	0.51	0.62	52.7	6.9	7.9	61.0
	2020	1.29	1.02	1.52	1.40	1.60	53.1	8.5	7.1	60.1
	2040	3.95	2.78	5.61	4.63	4.89	54.7	13.4	7.0	52.2
	2060	6.17	4.98	9.26	6.83	6.79	61.4	22.4	7.3	46.5
India	2008	0.23	0.23	0.21	0.21	0.25	32.3	5.3	5.7	27.0
	2020	0.50	0.42	0.52	0.51	0.61	32.0	5.9	4.6	27.4
	2040	2.02	1.51	2.46	2.23	2.51	35.4	7.5	4.9	29.9
	2060	4.79	4.07	6.09	5.04	5.28	39.6	12.1	4.8	29.4

^a in percent; ^b These are nominal (tax exclusive) rates. The effective (tax inclusive) taxrate (the nominal divided by 1 plus the nominal rate) is used in forming effective wage tax rates.

Table 4-5 reports the development of the macroeconomic variables as well as average effective wage tax rates in the baseline path from 2008 until 2060 for the five regions. All indexes for the five regions are expressed relative to year-2008 values in the U.S. Obviously, the U.S. economy grows much faster over the coming decades compared to the EMU and NEA. While U.S. GDP expands by a factor of 2.6, EMU GDP increases by a factor of only 2.1, and NEA GDP by a factor of 2.2 until 2060. These differences mainly reflect demographic differences across the regions, particularly the absolute population decline in the EMU and NEA.

However, due to the major productivity increases in China and India, today's developing regions become the largest economies by 2060. For example, China's GDP starts at 54 percent of the U.S. value, but overtakes the U.S. by 2020. In 2060, China's GDP exceeds the U.S. GDP by a factor of 2.4. Thanks to India's slower productivity growth, its GDP level in 2060 is only 78 percent of China's GDP. But nevertheless, India's GDP exceeds the U.S. level by a factor of 1.9. Together, China and India account for almost 70 percent of total 5-region GDP. In 2008, their share amounted to only 25 percent.

The growth of output can be explained, in part, by the growth of inputs as shown in table 4-5. Capital stocks in each region grow almost in lockstep with the respective GDP. Equilibrium labor demand (and supply) rise both because of technical progress (the expansion of effective time available to successive cohorts) and changes in labor supply. In the other regions, this growth also reflects changes in labor productivity as successive new cohorts of workers gradually attain U.S. productivity levels. Consequently, labor supply increases very rapidly in China between 2008 and 2040 and in India between 2040 and 2060.

Table 4-6 decomposes each region's GDP into its components. Over the first half of this century, the U.S. runs sizeable annual trade deficits as it exports capital to the fast growing Chinese and Indian economies. After 2040, capital is repatriated to the U.S., which explains why the U.S. trade deficit almost vanishes towards the year 2060 and why China runs such large trade deficits. Similar developments as in the U.S. can be observed for EMU and NEA. Both regions start out from trade surpluses which become smaller and even turn into deficits by mid-century. However, both regions also accumulate major trade surpluses by 2060. In contrast to China, India uses much more capital than it owns until the very end of the century. From table 4-6 it becomes obvious that trade and current account deficits can be very sizeable for decades before they reverse their sign and that their pattern is not easily forecast, i.e., there are a host of complex, interconnected factors determining their time paths including inter-regional differences in saving behavior, demographics, and fiscal policies.

Consider next how average effective wage tax rates evolve. As indicated in Table 4-5, this tax rate comprises the payroll tax rate, the average wage (labor income) tax rate, and the average

Table 4-6: GDP and its components in the baseline path

	Year	Consumption /GDP ^a	Investment /GDP ^a	Gov. Purchases /GDP ^a	Trade Balance /GDP ^a	Current Account /GDP ^a
U.S.	2008	72.2	20.6	16.2	-9.0	-0.3
	2020	76.9	15.0	17.6	-9.5	6.5
	2040	76.5	19.8	20.4	-16.7	0.8
	2060	57.3	22.4	20.7	-0.4	-2.3
EMU	2008	58.7	17.5	20.4	3.5	2.8
	2020	61.4	16.1	21.3	1.1	5.7
	2040	63.2	19.2	24.1	-6.5	2.0
	2060	51.1	22.0	25.6	2.3	0.4
NEA	2008	58.8	16.1	17.3	7.7	6.2
	2020	62.2	17.9	19.5	0.4	2.5
	2040	60.0	18.6	21.2	0.2	5.3
	2060	52.6	20.5	22.4	4.5	3.4
China	2008	38.5	25.2	29.0	7.2	-3.8
	2020	35.5	29.8	27.7	6.9	-5.2
	2040	42.2	21.8	31.2	4.9	2.4
	2060	49.5	21.2	35.7	-6.4	2.3
India	2008	59.9	26.8	18.0	-4.7	-11.9
	2020	49.2	32.8	16.5	1.5	-14.2
	2040	49.2	27.0	16.3	7.5	-8.3
	2060	49.8	25.5	17.9	6.7	-2.6

^a in percent

consumption tax rate.²³ All five regions experience dramatic increases over time in tax rates. In the U.S., average effective wage tax rates rise from 39.3 percent in 2008 to 60.5 percent in 2060. This is a larger percentage increase than in the EMU, which starts with a 56.2 percent average rate and ends up in 2060 with an 73.2 percent average rate. In NEA, the average effective wage tax rate rises from 60.4 percent to 74.1 percent.

These tax hikes can be explained as follows. First, all three regions have very significant pay-go social insurance programs whose benefits are disproportionately distributed to the elderly. Given the dramatic aging (see table 4-1) now underway, one would expect major tax increases simply to finance these benefits. Second, healthcare benefit levels have risen and can be expected to continue to rise much faster than per capita GDP in each of the three regions. Recall

²³Consumption tax rates are expressed here on a tax inclusive basis to make it comparable to payroll and wage tax rates.

that we are assuming for the U.S., EMU, and NEA a growth rate in healthcare benefit levels that is two percentage points higher than the growth rate of per capita GDP until 2025 and one percentage point higher for the following 10 years.

Tax rates in both China and India also end up much higher at the end of the century, but until 2040 they rise rather moderately. The explanation is the expansion of the labor income of younger generations thanks to our assumptions concerning cohort-specific labor-productivity increases. Over time, as the entire labor force becomes fully productive and as these fully productive generations retire, their higher earnings histories translate into higher old age healthcare and pension benefits, whose financing requires higher tax rates. As indicated, we also assume a very sizable growth rate in healthcare benefit levels (4 percentage points above the growth rate of per capita GDP) for the next four decades in these two regions. Our rationale is that these countries are starting out with very low levels of healthcare benefits; and their governments will face strong pressure from their populations to improve this situation.

Table 4-6 shows the compositions of the tax rate increases. In the three developed regions they are coming disproportionately from increases in payroll and consumption tax rates. In China and India the major increase is from payroll taxes. Long-run average wage tax rates (the rates applied to wages via income taxation) in both regions differ either not at all or very little from their initial values. In the case of China, long-run consumption tax rates are significantly lower than their initial values. The decline in consumption tax rates compared to wage tax rates reflects the expansion of the consumption tax base relative to the wage tax base. The explanation lies in the significant aging of the Chinese population, which generates relatively large numbers of elderly whose principal occupation is consuming, rather than working.

Changes in factor prices are shown in Table 4-7. The interest rate increases during the next three decades thanks to capital-shallowing due to the ongoing aging process of developed regions' populations. Afterwards, it decreases again but in 2060 ends up 200 basis points above the year-2008 value. Low-skilled wages per unit of effective time decrease continuously during the century. By 2060 the level of wages for low-skilled workers is only 65 percent of the year-2008 level. Wages per unit of effective time for medium-skilled workers decline only

Table 4-7: Factor prices in the baseline path

Year	Interest Rate	Wage Rates*		
		Low	Middle	High
2008	7.9	1.00	3.23	6.88
2020	10.7	0.86	2.97	6.29
2040	12.5	0.68	2.77	6.34
2060	9.9	0.65	2.92	7.19

* at age 21 per unit of effective time

moderately during the transition. The wage rate of high-skilled workers per unit of effective time also declines during the next decade. But it increases afterwards, so that in 2060 it is 4.5 percent above the 2008 level. Wage-skill differentials in the medium and long runs are much larger than those that now exist. In 2008 the high-skilled wage rate exceeds the low-skilled rate by a factor of 6.8. By 2060 this factor is 11.1.

Finally, it is interesting to have a closer look at the region-specific structure of production during the transition as reported in table 4-8. Due to increasing government expenditures over time, the share of public good production in overall production rises over time in all five regions. Since high-skilled labor is relatively abundant in the developed regions, production in these regions is mainly concentrated in the skill-intensive sectors – namely high-tech and investment good production. In contrast, production in the low-tech traded good sector is essentially eliminated in the medium run in the developed regions. In the context of these results, which permit offshoring, "essentially eliminated" refers to producing, with the help of overseas labor, just enough (as in essentially zero) of the low-tech traded good to permit factor price equalization.

In China and India, production of low-skill intensive goods (housing and the low-tech traded good) is significant, but the technological catch up process yields a very complex production pattern over time in these countries. The developed regions as well as India after 2020 become major importers of the low-tech consumption good, while China becomes the world's primary exporter of this good. The opposite holds for the high-tech good which is mainly imported by China and exported by the U.S. (by mid-century), EMU, NEA, and India. During the first two decades China and India are the main importers of the investment good while this good

Table 4-8: Production and trade balance

		Share of good in domestic production ^a				Trade balance ^b for				Offshore labor ^c			
		Ser- vices	Hou- sing	Low tech	High tech	Invest. Good	Public Good	Low tech	High tech	Invest. Good	low	middle	high
U.S.	2008	6.0	30.2	10.3	10.8	26.3	16.2	-1.2	-13.5	5.7	0.1	0.0	0.0
	2020	6.9	32.1	2.3	15.2	25.7	17.6	-9.9	-10.4	10.8	1.8	1.0	0.6
	2040	7.6	32.1	0.0	20.9	19.1	20.4	-12.1	-3.9	-0.7	-16.4	-6.8	-3.3
	2060	5.6	24.0	0.0	23.6	26.0	20.7	-9.1	5.0	3.6	-25.6	-10.1	-4.8
EMU	2008	5.1	24.6	2.5	25.9	21.4	20.4	-6.8	6.4	3.9	0.1	0.0	0.0
	2020	5.6	25.6	0.0	26.5	20.9	21.3	-9.8	6.1	4.8	-4.4	-1.7	-0.7
	2040	6.4	26.5	0.0	30.5	12.6	24.1	-10.0	10.1	-6.6	-20.4	-7.6	-3.0
	2060	5.0	21.4	0.0	29.4	18.6	25.6	-8.1	12.8	-2.4	-26.2	-9.1	-3.6
NEA	2008	5.6	24.7	0.0	28.3	24.0	17.3	-9.3	9.1	8.0	-0.5	-0.2	-0.1
	2020	6.3	26.1	0.0	25.0	23.1	19.5	-9.8	4.9	5.2	-10.9	-4.4	-2.1
	2040	6.2	25.2	0.0	22.8	24.6	21.2	-9.4	3.6	6.0	-22.6	-8.2	-3.8
	2060	5.5	22.1	0.0	19.0	31.0	22.4	-8.3	2.3	10.5	-28.6	-10.0	-4.8
China	2008	2.6	16.2	23.8	17.5	10.9	29.0	17.5	4.0	-14.3	0.1	0.0	0.0
	2020	2.1	14.7	22.9	11.7	20.9	27.7	17.0	-1.1	-9.0	3.1	1.3	0.6
	2040	2.6	17.7	17.2	6.8	24.4	31.2	10.3	-8.1	2.6	9.6	3.9	1.6
	2060	3.6	20.7	17.3	3.6	19.1	35.7	9.3	-13.6	-2.1	10.6	4.0	1.6
India	2008	4.1	25.1	16.1	30.5	6.2	18.0	6.4	9.6	-20.6	0.1	0.0	0.0
	2020	3.1	20.5	13.6	28.7	17.6	16.5	5.6	11.1	-15.2	2.8	1.3	0.8
	2040	2.8	20.6	6.6	29.3	24.4	16.3	-1.5	11.7	-2.6	9.0	4.5	2.6
	2060	3.2	20.8	5.6	27.2	25.1	17.9	-2.5	9.6	-0.4	10.7	5.4	3.0

^a in percent; ^b in percent of GDP; ^c in percent of domestic skill-specific labor demand

is exported by the developed regions. After 2020, the EMU also becomes a major importer of investment goods. Furthermore, all three developed regions become increasingly dependent over time on off-shore low-skilled labor. By mid-century about one fourth of the developed regions' low-skilled labor forces are hired off shore.

4.3 Simulation of unilateral policy reforms

In the following we consider various social security and tax reforms in order to analyze their impact on global macroeconomic developments and their distributional consequences. The scenarios include the privatization of public pension systems, the limitation of growth in healthcare benefits, the financing of public expenditures by debt and a hike in government expenditures that is financed either by taxes or by public debt. All scenarios are unanticipated and start in year 2009. We first simulate these different scenarios as an unilateral reform in the U.S. The rationale is to analyze the international spillover effects of uncoordinated politics. Afterwards, in section 4.4, the same scenarios are considered assuming a worldwide coordination of reforms.

4.3.1 Macroeconomic effects

The first policy reform assumes a gradual phase-out of the public pension system over a period of 40 years, i.e. pension replacement rates are linearly reduced to reach zero for new retirees from 2049 onwards. The simulation results are reported in the upper part of table 4-9. Not surprisingly, the privatization of the pension system in the U.S. leads to a major reduction of the payroll tax burden. In 2060, this decline is almost 15 percentage points. Lower tax burdens translate in higher capital formation. There is however only a minor increase in the capital stock until mid-century and in 2060 it is even 1 percent lower than in the baseline path. This seems surprising but can be explained when looking at the results of the other four regions. They obviously can increase their capital stocks over time since the higher capital supply of the U.S. is shared internationally. Capital mainly flows to the Asian economies so that in 2060 the capital stock in China is 6.6 percent and in NEA 5.9 percent above its respective baseline value. This also translates into higher GDP growth in the Asian regions and smaller long-run GDP in the U.S. Note also the changes in skill-specific labor demand. Since less capital is now employed in the U.S. production of traded goods switches to the investment good sector which

Table 4-9: Privatization of pension systems

	Year	GDP ^a	Capital Stock ^a	Labor Demand ^a			Effective Wage Tax ^b	Pay-roll Tax ^b	Average Wage Tax ^b	Consumption Tax ^b
				Low	Middle	High				
(a) Unilateral reform in the U.S.										
U.S.	2020	1.3	1.8	1.3	1.8	0.4	-2.2	-2.2	-0.1	0.1
	2040	1.6	2.9	5.6	4.6	-3.7	-11.3	-10.2	-0.6	-0.6
	2060	-4.1	-1.0	2.2	-1.4	-11.5	-18.2	-14.8	-1.0	-3.3
EMU	2020	0.6	1.6	0.0	0.1	0.1	-0.1	-0.1	-0.1	0.1
	2040	1.7	4.0	-0.6	0.3	0.6	0.1	-0.4	0.0	0.7
	2060	2.6	5.8	-0.6	0.5	1.1	1.0	-0.4	0.0	1.7
NEA	2020	0.6	1.5	0.0	0.1	0.1	-0.3	-0.1	-0.2	0.0
	2040	1.6	4.0	-0.7	0.1	0.5	-0.6	-0.3	-0.4	0.2
	2060	2.6	5.9	-0.7	0.3	1.4	-0.3	-0.4	-0.4	0.9
China	2020	0.4	1.4	0.0	0.0	-0.2	0.0	-0.1	0.0	0.2
	2040	1.5	3.9	-0.6	0.2	0.3	0.5	0.0	0.0	1.3
	2060	3.7	6.6	0.3	1.9	2.6	1.5	-0.1	0.1	3.3
India	2020	0.5	1.5	0.0	0.0	-0.2	-0.1	0.0	0.0	-0.2
	2040	1.0	3.6	-0.7	-0.4	-0.5	-0.2	0.0	-0.1	-0.2
	2060	2.1	5.4	-0.3	0.2	0.3	0.2	0.0	-0.1	0.4
(b) Multilateral reform										
U.S.	2020	2.8	4.9	1.3	2.4	1.4	-2.5	-2.5	-0.2	0.3
	2040	7.1	13.4	6.7	7.1	0.3	-10.7	-10.7	-0.5	0.7
	2060	4.7	17.5	3.0	1.6	-5.7	-16.2	-14.9	-3.8	-0.5
EMU	2020	2.6	4.9	1.7	1.9	0.9	-2.7	-2.4	-0.4	0.1
	2040	4.5	11.5	3.8	3.5	-2.0	-13.0	-10.4	-1.5	-1.6
	2060	2.9	16.2	1.3	-0.7	-7.2	-19.0	-14.1	-2.3	-4.8
NEA	2020	2.0	4.5	0.8	1.0	0.5	-2.4	-2.1	-0.4	0.1
	2040	4.3	11.9	0.9	1.3	-0.7	-11.3	-9.4	-1.5	-0.5
	2060	5.1	19.1	-1.7	-1.5	-2.3	-15.2	-12.6	-1.9	-1.1
China	2020	1.2	3.8	-0.2	-0.2	-0.4	-0.3	-0.6	-0.1	1.0
	2040	4.6	12.3	-0.8	0.5	0.9	0.4	-1.1	0.0	3.6
	2060	12.1	24.9	1.8	5.9	6.2	-1.3	-5.4	0.0	9.2
India	2020	1.2	3.9	-0.2	-0.2	-0.4	-0.8	-0.5	-0.1	-0.4
	2040	3.2	11.3	-1.4	-0.9	-1.3	-1.8	-1.0	-0.3	-0.9
	2060	7.1	21.2	-0.3	0.5	-0.1	-3.2	-2.8	-0.4	-0.1

Changes to the respective baseline values in ^a percent or ^b percentage points.

is less capital and high-skilled labor intensive but uses more low- and medium-skilled labor. As a consequence, labor demand for high-skilled workers in the U.S. is reduced and increases for the other both skill classes. The exactly opposite happens in the other regions especially in

China.

Higher worldwide capital supply lowers the interest rate during the transition path as shown in table 4-10 while wage rates are improved. The effects on low- and medium-skilled wages are rather moderate. However, high-skilled wages are increased by 2.7 percent in 2060.

Table 4-10: Factor prices in alternative policy paths

Policy	Year	Interest	Wage Rates ^b			Interest	Wage Rates ^b		
		Rate ^a	Low	Middle	High	Rate ^a	Low	Middle	High
		(a) Unilateral reform in the U.S.				(b) Multilateral reform			
Privatization of pension systems	2020	-0.2	0.3	0.4	0.8	-0.5	1.1	1.4	1.8
	2040	-0.5	0.8	0.9	2.1	-1.5	3.0	3.6	5.4
	2060	-0.5	1.1	1.3	2.7	-2.1	5.5	6.6	8.7
Limit growth in healthcare benefits	2020	0.0	0.1	-0.1	-0.4	0.1	0.4	-0.2	-0.6
	2040	0.0	0.6	0.2	-0.2	-0.1	2.6	0.5	-0.4
	2060	0.0	0.7	0.4	0.0	-0.6	4.8	2.2	1.4
Debt financing	2020	0.1	-0.1	-0.4	-0.3	0.0	0.0	-0.1	-0.1
	2040	0.2	-0.7	-0.5	-0.7	0.0	-0.1	0.2	-0.1
	2060	0.3	-0.7	-0.9	-0.9	0.1	0.0	-0.1	-0.4
Hike in public expenditures / Taxes	2020	0.0	-0.6	-0.3	0.2	0.0	-3.1	-1.0	1.8
	2040	0.0	-0.4	-0.2	0.2	0.2	-3.7	-1.5	1.8
	2060	0.1	-0.3	-0.2	0.2	0.2	-3.7	-1.6	2.0
Hike in public expenditures / Debt	2020	0.0	-0.8	-0.2	0.3	0.4	-4.7	-2.4	0.6
	2040	0.3	-0.9	-0.8	-0.5	1.5	-6.6	-4.7	-1.6
	2060	0.2	-0.7	-0.8	-0.4	1.2	-6.4	-5.0	-0.6

Changes to the respective baseline values in ^a percentage points or ^b percent.

Now we turn to the scenario where growth in healthcare benefits is limited. To be precise, we assume that government healthcare benefits are prevented from growing at higher rates than per capita GDP over the next decades. As table 4-11 suggests there are only modest effects on the economic development in the U.S. Effective wage tax rates are reduced compared to the baseline path. In 2060, this decline is 7.8 percentage points. The lower tax rates don't manifest themselves in higher GDPs. The reason is income effects on labor supply; agents in our model respond to their tax cuts by consuming more leisure, not working harder, which leaves aggregate output little changed from the baseline case. Obviously, factor prices are hardly changed in this unilateral policy reform (see table 4-10). Since there are only minor

Table 4-11: Limiting growth in healthcare benefits

	Year	GDP ^a	Capital Stock ^a	Labor Demand ^a			Effective Wage Tax ^b	Pay-roll Tax ^b	Average Wage Tax ^b	Consumption Tax ^b
				Low	Middle	High				
(a) Unilateral reform in the U.S.										
U.S.	2020	-0.3	0.1	0.1	-0.4	-0.9	-2.9	-0.7	-1.2	-1.2
	2040	0.8	2.6	1.8	0.3	-1.4	-6.9	-1.7	-3.0	-2.7
	2060	-1.4	1.2	0.2	-2.2	-3.9	-7.8	-1.3	-3.2	-4.6
(b) Multilateral reform										
U.S.	2020	-0.5	-0.2	-0.1	-0.6	-1.1	-2.8	-0.6	-1.2	-1.3
	2040	0.0	2.3	-0.7	-0.9	-2.1	-7.2	-1.7	-3.2	-2.9
	2060	0.2	6.1	-3.1	-2.7	-3.2	-8.4	-1.8	-3.5	-4.3
EMU	2020	-0.9	-0.8	0.3	-0.3	-1.8	-3.4	-2.6	-0.2	-0.9
	2040	-1.2	0.6	0.5	-0.5	-4.2	-8.3	-6.3	-0.5	-2.2
	2060	-1.4	4.1	-2.6	-3.0	-5.7	-8.9	-6.5	-0.7	-3.4
NEA	2020	-0.7	-0.7	0.0	-0.4	-1.3	-3.0	-2.5	-0.1	-0.7
	2040	-1.2	0.7	-1.3	-1.5	-3.4	-7.7	-5.6	-0.6	-2.3
	2060	-0.8	4.9	-4.2	-3.6	-4.0	-9.4	-5.9	-1.3	-3.5
China	2020	-0.8	-0.8	-0.4	-0.8	-0.9	-2.3	-1.8	0.0	-1.4
	2040	-1.6	1.0	-1.2	-2.2	-4.5	-9.5	-8.0	0.1	-3.7
	2060	-1.3	5.4	-0.9	-2.4	-8.3	-14.8	-13.2	0.2	-3.9
India	2020	-0.4	-0.6	-0.1	-0.2	-0.3	-1.1	-1.0	0.0	-0.1
	2040	-0.3	1.3	-0.4	-0.9	-1.8	-4.9	-4.2	0.0	-1.2
	2060	1.1	6.6	-0.1	-1.0	-3.2	-8.3	-7.2	-0.1	-1.6

Changes to the respective baseline values in ^a percent or ^b percentage points.

international spillover effects, we do not report the simulation results for the other regions.

The next three scenarios are dedicated to tax policy. The first scenario assumes that public expenditures are financed by endogenous public debt over a period of 20 years instead of endogenously adjusting wage and consumption tax rates as in the baseline path. Thus the intercept and the progressive term of the wage tax formula as well as the consumption tax rates are frozen at their year-2008 level until 2028 and are endogenously adjusted again from year 2029 onwards to balance the government's budget. During the period 2009 and 2028 the public deficit is calculated endogenously given tax revenues. The new debt to output ratio reached at the beginning of year 2029 is kept fixed over the remaining years of the transition.

When government expenditures are financed by public debt, U.S. debt increases from initially

Table 4-12: Debt financing of public expenditures

	Year	GDP ^a	Capital Stock ^a	Labor Demand ^a			Effective Wage Tax ^b	Pay-roll Tax ^b	Average Wage Tax ^b	Consumption Tax ^b
				Low	Middle	High				
(a) Unilateral reform in the U.S.										
U.S.	2020	0.6	0.0	0.6	1.1	0.9	-3.3	-0.1	-2.2	-1.2
	2040	-4.6	-5.0	-5.3	-5.7	-3.1	10.1	0.6	6.3	4.4
	2060	2.6	0.2	1.9	3.6	4.8	9.5	-1.1	5.6	7.6
(b) Multilateral reform										
U.S.	2020	0.9	0.6	0.6	1.2	1.1	-3.4	-0.2	-2.2	-1.2
	2040	-3.2	-2.8	-4.5	-4.5	-2.0	9.9	0.3	6.2	4.7
	2060	3.8	2.4	2.1	4.1	5.8	9.3	-1.2	5.3	8.0
EMU	2020	-0.3	-0.4	-0.3	-0.2	-0.1	0.1	0.0	-0.4	0.7
	2040	0.3	0.3	0.8	0.4	0.2	-0.7	0.1	-0.3	-0.7
	2060	-0.8	-1.1	-0.5	-0.7	-0.7	-0.6	0.1	-0.4	-1.2
NEA	2020	-0.1	-0.3	-0.1	0.1	0.0	-0.3	0.0	-0.9	0.8
	2040	-0.3	-0.3	-0.1	-0.4	-0.2	1.9	0.1	1.0	1.3
	2060	0.1	-0.4	0.1	0.3	0.4	1.5	-0.1	0.8	1.4
China	2020	-0.1	-0.3	0.1	0.0	0.0	1.6	0.0	1.2	0.9
	2040	0.1	0.2	0.5	0.1	0.0	-2.6	0.1	-0.5	-4.8
	2060	-1.6	-1.5	-0.7	-1.5	-1.8	-2.8	0.1	-0.5	-5.1
India	2020	-0.1	-0.2	0.1	0.0	0.0	1.0	0.0	1.2	-0.4
	2040	0.2	0.2	0.5	0.2	0.1	-0.8	0.0	-0.2	-1.0
	2060	-0.6	-0.9	-0.3	-0.5	-0.5	-1.0	0.0	-0.2	-1.4

Changes to the respective baseline values in ^a percent or ^b percentage points.

70 percent of GDP to 153 percent from 2029 onwards. Higher debt implies much higher wage and consumption tax rates in the medium and long run in order to finance increased interest payments. As reported in table 4-12, the average wage tax rate increases by 5.6 percentage points and the consumption tax rate by 7.6 percentage points in 2060 compared to their respective baseline values. As one would have expected, this development leads to lower capital and labor demand and thus smaller GDP in the medium run. In 2040, the U.S. capital stock is 5 percent below the respective baseline value and GDP is reduced by 4.6 percent. However, in the longer run the capital stock is hardly changed compared to the baseline path. The reason is that less capital is invested abroad in order to pay for the higher debt burden. Consequently, capital stocks in the other four regions decline with respect to their baseline values. This de-

velopment increases U.S. labor demand and GDP, which improves by 2.6 percent compared to the year-2060 baseline level. The smaller worldwide capital supply increases the medium- and long-run interest rate and decreases the wage rates for all three skill classes accordingly (see table 4-10).

The final scenarios are motivated by recent government policies to deal with the financial and economic crisis. In order to stabilize the economy all developed countries increased their government expenditures dramatically. The question of course is how these hikes in government expenses should be financed in the medium and long term. One way is to raise general taxes. The other way that most countries are currently following is to increase public debt. Of course, both alternatives lead to different macroeconomic and distributional effects.

In the following, we therefore analyze how the different alternatives to finance the hike in public expenditures affect the economic development in our globalized world. Precisely, we assume an immediate increase of government expenditures (except the share of health benefits financed by general tax revenues) by 20 percent which is maintained during the whole transition path. We then distinguish two options to finance the higher expenses. First, they are financed by letting endogenous (wage and consumption) taxes balance the government budget. Second, government expenditures are financed by public debt over a period of 10 years while wage and consumption tax rates are fixed during this time and adjust endogenously again from 2019 onwards.

Consider first the closure of the U.S. government's budget by taxes as shown in table 4-13. The 20-percent increase of education and remaining government expenditures means a sudden increase in overall expenditures in the U.S. by 2.1 percent of GDP. In 2060, U.S. government expenditures amount to 22.7 percent of GDP instead of 20.7 percent as in the baseline. Average wage tax rates in the U.S. are raised by 1.6 percentage points during the baseline. The consumption tax rate in 2060 is 2.5 percentage points above its baseline value. The effects on the capital stock, labor demand and GDP are rather small. Also factor prices remain fairly unchanged in this reform scenario.

The effects of the increase in government expenditures on the economic developments in the

Table 4-13: Hike in government expenditures financed by taxes

	Year	GDP ^a	Capital Stock ^a	Labor Demand ^a			Effective Wage Tax ^b	Pay-roll Tax ^b	Average Wage Tax ^b	Consumption Tax ^b
				Low	Middle	High				
(a) Unilateral reform in the U.S.										
U.S.	2020	0.4	-0.8	0.7	1.1	1.3	2.8	-0.2	1.7	1.6
	2040	0.5	-0.7	0.1	1.2	1.6	2.4	-0.4	1.6	1.7
	2060	0.7	-0.5	-0.1	1.1	1.8	3.1	-0.2	1.6	2.5
(b) Multilateral reform										
U.S.	2020	0.6	-0.6	1.0	1.3	1.2	2.7	-0.3	1.7	1.7
	2040	0.7	-0.9	2.6	1.9	1.0	2.9	-0.2	1.8	1.8
	2060	0.9	-0.7	2.5	2.0	0.8	3.4	-0.1	1.8	2.5
EMU	2020	1.2	-0.3	1.9	2.0	1.8	4.2	-0.2	1.6	4.3
	2040	0.8	-1.3	2.0	2.0	1.3	4.3	-0.1	1.7	4.1
	2060	0.9	-1.2	1.9	2.1	1.2	5.2	-0.2	1.8	5.5
NEA	2020	0.9	-0.4	1.9	1.7	1.4	4.1	-0.2	1.9	3.7
	2040	0.9	-1.1	2.5	2.2	1.3	4.3	-0.1	2.0	4.0
	2060	1.0	-1.0	2.6	2.4	1.2	4.6	-0.1	2.0	4.7
China	2020	0.5	-2.4	1.0	2.0	2.3	8.5	-0.1	1.2	21.5
	2040	1.8	-1.9	2.0	3.8	3.9	8.0	-0.1	1.3	17.6
	2060	2.7	-1.4	2.7	5.1	5.0	7.9	-0.3	1.4	16.1
India	2030	-0.3	-1.9	0.1	0.5	0.5	4.8	-0.1	0.8	7.0
	2050	0.0	-2.0	0.5	1.1	0.9	4.2	-0.1	0.7	6.4
	2100	0.4	-1.6	0.9	1.6	1.0	4.2	-0.1	0.7	6.3

Changes to the respective baseline values in ^a percent or ^b percentage points.

U.S. are however much larger if government expenditures are financed by public debt over a period of 10 years (see table 4-14). Debt now increases to 121 percent of GDP. This increase in public debt of about 50 percentage points over time is in line with recent projections for U.S. federal debt in 2020 by the CBO (2011). Of course, in order to finance higher interest payments taxes have to increase in the medium and long run. Thus, in 2060 the average wage tax rate is now 5.2 percentage points above its respective baseline value and the consumption tax rate is increased by 8 percentage points. The macroeconomic effects are very similar to those shown in the scenario where the budget was financed by public debt without assuming a hike in government expenditures. The large tax increases lead to a reduced capital stock, labor demand and GDP in the medium run. But after 2040 the capital stock recovers and labor

Table 4-14: Hike in government expenditures financed by debt

	Year	GDP ^a	Capital Stock ^a	Labor Demand ^a			Effective Wage Tax ^b	Pay-roll Tax ^b	Average Wage Tax ^b	Consumption Tax ^b
				Low	Middle	High				
(a) Unilateral reform in the U.S.										
U.S.	2020	-2.8	-3.2	-1.5	-2.8	-2.3	9.0	0.4	5.4	4.2
	2040	-0.9	-3.1	-1.6	-0.3	1.5	9.0	-0.4	5.7	5.1
	2060	3.1	0.3	1.6	4.2	5.9	9.4	-1.0	5.2	8.0
(b) Multilateral reform										
U.S.	2020	-5.1	-7.4	-1.5	-4.1	-4.3	10.2	0.8	6.2	4.2
	2040	-6.5	-12.0	-2.0	-4.2	-3.4	10.0	0.9	6.5	3.6
	2060	-1.5	-8.2	3.5	2.8	0.7	9.1	-0.8	6.1	5.8
EMU	2020	-2.1	-5.5	0.2	-0.5	-0.5	10.2	0.6	4.3	8.5
	2040	-3.7	-10.7	1.3	0.2	-0.6	11.1	1.2	5.0	7.7
	2060	-1.3	-8.6	3.6	3.7	1.0	10.2	-0.2	4.5	9.9
NEA	2020	-1.9	-5.3	1.0	-0.1	-0.6	11.3	0.8	5.6	7.7
	2040	-3.2	-10.3	2.1	1.1	0.1	13.8	0.9	7.3	9.6
	2060	0.3	-7.4	6.1	6.1	2.6	12.5	-0.2	6.4	11.3
China	2020	-0.4	-5.6	1.2	2.4	2.8	10.0	-0.1	1.7	24.7
	2040	-0.9	-10.2	3.0	4.6	4.4	13.5	-0.1	2.8	30.0
	2060	-1.0	-9.8	2.5	4.6	3.5	11.7	-0.4	2.7	23.3
India	2020	-1.4	-5.4	0.2	0.7	0.9	6.3	0.0	1.2	8.9
	2040	-2.4	-10.1	1.3	2.0	1.8	8.4	0.0	1.6	12.6
	2060	-2.4	-9.4	0.8	1.7	0.8	7.5	-0.2	1.6	11.1

Changes to the respective baseline values in ^a percent or ^b percentage points.

demand exceeds the baseline values in all skill levels. Consequently, the U.S. GDP level is increased by 3.1 percent in 2060. The reason for this development is that less capital flows from the U.S. to the emerging markets. The lower capital formation in the U.S. means a decrease in worldwide capital supply so that the interest rate is slightly increased during the transition path as reported in table 4-10. Consequently, the wage rates of all three income classes are suppressed slightly.

4.3.2 Welfare effects

Table 4-15 shows the welfare effects of the unilateral policy reforms in the U.S. Since all scenarios led to rather small welfare changes in the other four regions we only report the values for the U.S. but discuss the effects for the other regions when necessary. Welfare changes in

this section are measured as Hicksian equivalent variation and expressed as a percentage of remaining lifetime resources.

Consider first the privatization of the U.S. pension system. Obviously, initially older and middle-aged agents in the U.S. are experiencing small welfare losses which is mainly due to the lower generosity of the pension system. However, young and future cohorts are experiencing large welfare gains since they benefit from the reduced payroll tax burden and higher wage rates. These welfare gains can be very sizeable. Take e.g. low-skilled workers that are born in 2040. They experience a welfare gain of 23.1 percent of their lifetime resources. Note that the members of the high-income class experience smaller welfare gains than those of the low- and medium-income class. The reason is the ceiling on key payroll taxes that limits the benefits they can expect from cuts in these taxes. Welfare gains are highest for people in the low-income class since they enjoy less leisure and thus work more than their counterparts in the middle- and high-income class. Consequently, they benefit more from the reductions in payroll tax rates. Welfare effects in the other regions are small but not negligible. Interestingly, agents in the EU, NEA and India experience small welfare improvements which are mainly due to the higher remuneration of labor. Chinese people, however, experience small welfare losses due to increases in the consumption tax rate.

When growth rates of U.S. healthcare benefits are limited (see 4-15), young and future cohorts again experience welfare gains which are due to the reductions in payroll tax rates. The distributional effects in this scenario are less sizeable than in the privatization scenario since tax rates decline to a smaller extent. However, future cohorts born in 2060 nevertheless are expected to experience sizeable welfare gains between 5 and 9 percent of their lifetime resources. People in the other regions are expected to experience small improvements in welfare.

As one would have expected, if government expenditures are financed by government debt during the first 20 years of the transition older and middle-aged cohorts in the U.S. will benefit. The reason is of course the reduced tax burden due to frozen wage and consumption tax rates. However, the need to raise wage and consumption taxes in the future to pay for the higher interest on public debt leads to welfare losses for future cohorts. These are very sizeable. Take

Table 4-15: Welfare effects of unilateral reforms in the U.S.

Birth Year	(a) Privatization of pension system			(b) Limit growth in healthcare benefits			(c) Debt financing		
	Income Class 1	Income Class 2	Income Class 3	Income Class 1	Income Class 2	Income Class 3	Income Class 1	Income Class 2	Income Class 3
1940	0.04	-0.07	-0.05	-1.29	-0.05	0.05	0.46	0.25	0.12
1960	-3.56	-1.83	-1.01	0.06	0.31	0.22	0.44	0.23	0.11
1980	-0.08	-0.05	-0.05	1.25	1.08	0.76	0.58	0.51	0.51
2000	6.45	4.76	3.87	4.32	3.56	2.64	-2.67	-2.05	-1.43
2020	16.94	12.17	9.38	6.13	5.28	3.93	-9.22	-7.35	-5.68
2040	23.08	16.60	12.18	7.37	6.21	4.53	-9.76	-7.95	-6.01
2060	30.18	22.03	15.88	8.87	7.43	5.41	-12.86	-10.52	-8.05

Birth Year	(d) Hike in government expenditures / Taxes			(e) Hike in government expenditures / Debt		
	Income Class 1	Income Class 2	Income Class 3	Income Class 1	Income Class 2	Income Class 3
1940	-0.92	-0.45	-0.21	-0.53	-0.18	-0.07
1960	-1.61	-0.78	-0.37	-0.79	-0.32	-0.14
1980	-2.26	-1.55	-1.01	-1.87	-1.20	-0.58
2000	-2.77	-2.02	-1.38	-8.87	-6.72	-5.12
2020	-3.04	-2.29	-1.61	-8.63	-6.86	-5.13
2040	-3.30	-2.51	-1.77	-9.85	-7.87	-5.85
2060	-3.50	-2.71	-1.94	-11.42	-9.24	-6.96

e.g. members of the low-skill class who will be born in 2060. They experience a decline of 12.9 percent of their lifetime resources. Distributional effects in the other regions are negligible.

A hike in government expenditures leads to welfare losses for the U.S. population. This is not surprising since the population has to pay for these additional expenses. Note however the different sizes of welfare losses in case of tax or debt financing. When additional government consumption is financed by taxes, younger and future cohorts are hurt almost the same. They experience welfare losses of about 2 to 3 percent of their lifetime income. When expenses are financed by debt future cohorts are hurt by much more. Low-skilled workers are expected to lose 9 percent of the lifetime resources, high-skilled workers about 5 percent. The reason is of course that the higher interest burden demands a larger rise in tax rates. Furthermore, there is a downward pressure on future wages. Consequently, the populations in the other regions also experience some small welfare losses.

4.4 Simulation of multilateral policy reforms

We now consider the same five policy scenarios as in section 4.3. The setting of the scenarios is exactly the same as before. The only difference is that all reforms are coordinated between the five regions and are processed in all regions at the same time.

4.4.1 Macroeconomic effects

The lower part of table 4-9 reports the simulation results of the multilateral privatization of public pension systems. In all regions, payroll tax rates are lowered to a large extent. The lower tax burden leads to increased capital formation and thus higher capital stocks in all five regions. Labor demand declines in the developed regions due to income effects on labor supply. While experiencing a decline in GDP in the unilateral reform scenario, the U.S. and also the remaining four regions experience major increases in their GDP compared to the baseline. In 2060, GDP exceeds its respective baseline value by 4.7 percent in the U.S., by 2.9 percent in the EMU, by 5.1 percent in NEA, by 12.1 percent in China and by 7.1 percent in India. The coordinated privatization of public pension systems has a huge impact on factor prices as shown in table 4-10. The long-run interest rate is now 210 basis points below the year-2060 baseline value. Wage rates are much higher during the transition. In 2060, low-skilled wages are increased by 5.5 percent and high-skilled wages by 8.7 percent compared to the baseline.

A very similar direction of effects is observed when growth in healthcare benefits is limited in all five regions at the same time (see table 4-11). Due to reduced tax burdens capital stocks are increased in all regions over time. However, this does not result in higher GDP levels due to reduced labor supply. The only exception is India where GDP in the longer run is slightly above its baseline value. The reason is that the decline in labor demand is rather moderate due to the ongoing productivity improvements. The higher capital stock thus overcompensates reduced labor demand and, as a consequence, GDP is slightly increased. Again, table 4-10 reports a decline in the worldwide interest rate during the transition path and increased wage rates for all three skill classes. The effects are however much more moderate than in the pension privatization scenario.

Consider now the scenario when public expenditures are financed by debt during the first 20

years of the transition, see table 4-12. When wage and consumption tax rates are fixed at their year-2008 level, public debt in the U.S. increases as before from 70 percent of GDP to 152 percent from year 2029 onwards. In NEA public debt rises from initially 146 percent to 158 percent over the same period. However, all other regions can reduce their debt levels: In the EMU there is a moderate decline from 76 percent to 69 percent and in India from 72 percent to 63 percent. Thanks to major reductions of wage and consumption tax rates due to enlargements in the respective tax bases during the first decades of the baseline transition path, China's initial debt-to-GDP ratio of 21 percent even turns into a surplus of 3.6 percent from 2029 onwards. The moderate effects on public debt in EMU, NEA and India lead to only minor changes in their macroeconomic variables. However, similar to the unilateral scenario, the U.S. capital stock is reduced in the medium run. The effect is now smaller than in the unilateral case. In the longer run, the capital stock increases to a larger extent in the multilateral reform and thus also GDP increases by more. The reason is that the U.S. economy now benefits from China's surplus. The effects of this scenario on factor prices are rather small. The reason is that due to increased public debt in the U.S. and NEA on the one hand and reduced debt in the other regions these opposing effects are ruled out on worldwide factor prices.

Now we turn to the scenarios which assume a hike in government expenditures. This scenario leads to an increase in public expenditures by 2.1 percentage points in the U.S., 3.1 percentage points in the EMU, 2.8 percentage points in NEA, 5.6 percentage points in China, and 2.8 percentage points in India in 2060 compared to the baseline. When these increases are financed by taxes (see table 4-13), wage and consumption tax rates are raised in all regions. Of course, since this effect is largest in China since the increase in government expenditures is the highest among all five regions. The higher tax burden reduces capital stocks in all regions. GDP levels are however increased due to higher labor supply. As the results in table 4-10 indicate, wage dispersion increases over time since wage rates for low-skilled workers are reduced and those for high-skilled workers increase. The reason is that low-skilled labor demand is raised by disproportionately more than high-skilled labor. The lower worldwide capital supply leads to small increases in the interest rate.

Finally, if the hike in government expenditures is financed by public debt, public debt has to

increase in all five regions. Over the first 10 years of the transition path debt levels rise to 125 percent in the U.S. and EMU, to 198 percent in NEA, to 77 percent in China, and to 105 percent in India. Of course, due to the higher future tax burdens capital stocks decrease over time. The effects are very sizeable. In 2060, the capital stock is reduced by 8.2 percent in the U.S., by 8.6 percent in the EMU, by 7.4 percent in NEA, by 9.8 percent in China and by 9.4 percent in India compared to their respective baseline values. Despite the improvements in labor supply, GDP levels in all five regions are smaller than in the baseline. Note that in the unilateral scenario long-run U.S. GDP was improved thanks to international spillovers. The multilateral scenario makes clear that simultaneous debt increases dampen worldwide macroeconomic growth. As expected, this scenario reveals major increases in the world interest rate. In 2060, it is 120 basis points above its baseline value. Wages are lowered for all three skill groups. Low-skilled wages are reduced by 6.4 percent, medium-skilled wages by 5.0 percent and high-skilled wages by 0.6 percent in 2060 (see table 4-10).

4.4.2 Welfare effects

Table 4-16 reports the welfare effects of the multilateral policy reforms. As one would have expected, the welfare changes in the privatization scenario are similar in all countries as they were in the unilateral reform in the U.S. The lower future tax burden and increased wage rates translate into sizeable welfare gains for all young and future cohorts in all regions. These gains are higher in the developed regions since their pension systems are much more generous than in China and India so that the payroll tax burden is lowered to a much larger extent over time.

Table 4-16: Welfare effects of multilateral reforms

(a) Privatization of pension systems

Birth Year	U.S. Income Class			EU Income Class			Japan Income Class		
	1	2	3	1	2	3	1	2	3
1940	0.11	-0.18	-0.13	-0.07	-0.22	-0.21	-0.22	-0.36	-0.30
1960	-3.77	-2.31	-1.49	-4.66	-3.39	-2.34	-4.11	-3.05	-1.96
1980	-0.37	-0.47	-0.61	-0.75	-0.59	-0.53	-0.23	-0.17	-0.41
2000	6.24	4.40	3.38	6.36	5.62	3.11	6.73	5.15	1.82
2020	17.29	11.69	8.58	16.16	13.66	10.33	16.73	13.44	6.35
2040	23.16	15.60	10.93	20.70	17.03	12.93	19.34	14.84	8.57
2060	30.57	21.05	14.52	22.74	18.67	13.45	20.79	15.19	8.27

	China Income Class			India Income Class		
	1	2	3	1	2	3
1940	-0.53	-0.74	-0.75	-0.35	-0.41	-0.40
1960	-3.01	-3.68	-3.68	-2.78	-2.95	-2.92
1980	0.34	-0.56	-0.61	0.46	0.10	0.16
2000	-1.10	-0.84	-0.64	0.85	1.00	1.44
2020	0.57	0.13	0.23	3.34	2.54	2.39
2040	7.66	4.91	3.57	7.75	5.48	4.22
2060	12.28	8.22	5.92	10.87	7.31	5.10

(b) Limiting growth in healthcare benefits

Birth Year	U.S. Income Class			EU Income Class			Japan Income Class		
	1	2	3	1	2	3	1	2	3
1940	-1.30	-0.04	0.06	-0.83	-0.04	0.09	-0.99	-0.05	0.11
1960	0.07	0.32	0.24	-0.13	0.35	0.34	-0.15	0.43	0.41
1980	1.36	1.06	0.72	1.35	1.52	1.28	1.68	1.71	1.15
2000	5.10	3.60	2.49	5.83	5.04	2.74	7.54	5.34	1.94
2020	8.71	5.91	4.01	8.46	7.41	5.13	11.77	8.56	3.82
2040	10.67	7.16	4.69	8.91	7.92	5.53	12.31	8.76	4.42
2060	12.31	8.39	5.46	9.02	8.18	5.43	12.50	8.86	4.21

	China Income Class			India Income Class		
	1	2	3	1	2	3
1940	-3.02	-0.78	-0.20	-0.92	-0.22	-0.05
1960	-4.13	-0.62	0.12	-1.76	-0.25	0.04
1980	-1.55	0.60	0.89	-1.09	0.06	0.17
2000	5.93	3.83	2.15	3.07	2.16	1.36
2020	14.78	9.48	6.73	8.40	5.38	3.78
2040	16.75	10.89	7.62	10.53	6.37	4.21
2060	17.88	11.90	8.28	10.59	6.08	3.78

Table 4-16 continued

(c) Debt financing

Birth Year	U.S. Income Class			EU Income Class			Japan Income Class		
	1	2	3	1	2	3	1	2	3
1940	0.48	0.22	0.10	-0.24	-0.15	-0.08	-0.20	-0.14	-0.08
1960	0.42	0.13	0.00	-0.11	-0.09	-0.06	-0.01	-0.04	-0.05
1980	0.57	0.45	0.38	0.04	0.03	0.02	0.32	0.27	0.17
2000	-2.46	-1.92	-1.43	0.62	0.57	0.31	0.67	0.54	0.27
2020	-8.68	-7.00	-5.57	0.69	0.62	0.41	-1.84	-1.47	-0.91
2040	-9.43	-7.75	-6.06	0.92	0.76	0.51	-1.46	-1.22	-0.94
2060	-12.30	-10.14	-7.97	1.06	0.85	0.55	-1.58	-1.31	-0.99
	China Income Class			India Income Class					
	1	2	3	1	2	3			
1940	-0.65	-0.62	-0.60	0.00	0.01	0.00			
1960	-0.68	-0.69	-0.63	-0.26	-0.29	-0.26			
1980	-0.92	-0.91	-0.85	-0.50	-0.51	-0.50			
2000	0.18	0.18	0.09	0.24	0.28	0.17			
2020	3.13	2.34	1.66	0.88	0.72	0.45			
2040	2.81	2.02	1.40	1.03	0.69	0.42			
2060	3.51	2.54	1.80	1.13	0.73	0.44			

(d) Hike in government expenditures / Taxes

Birth Year	U.S. Income Class			EU Income Class			Japan Income Class		
	1	2	3	1	2	3	1	2	3
1940	-0.97	-0.51	-0.24	-2.55	-1.79	-1.05	-2.32	-1.75	-1.06
1960	-2.35	-0.90	-0.37	-3.53	-2.14	-1.23	-3.87	-2.41	-1.23
1980	-3.47	-1.83	-0.68	-4.59	-3.09	-1.61	-5.09	-3.49	-1.69
2000	-4.86	-2.71	-0.98	-5.77	-3.74	-1.43	-6.64	-4.37	-1.43
2020	-5.39	-3.19	-1.14	-6.24	-4.30	-1.77	-7.28	-4.79	-1.70
2040	-5.47	-3.31	-1.14	-6.46	-4.52	-1.93	-7.01	-4.55	-1.64
2060	-5.74	-3.59	-1.27	-6.54	-4.62	-1.92	-6.87	-4.41	-1.51
	China Income Class			India Income Class					
	1	2	3	1	2	3			
1940	-10.93	-10.78	-10.43	-4.72	-4.61	-4.41			
1960	-12.26	-10.39	-8.33	-6.15	-4.82	-3.41			
1980	-13.21	-11.24	-8.92	-7.16	-5.52	-3.54			
2000	-11.76	-7.96	-4.84	-7.68	-5.11	-2.49			
2020	-11.38	-7.41	-4.22	-7.19	-4.36	-1.74			
2040	-10.74	-6.93	-3.77	-6.59	-3.73	-1.28			
2060	-10.77	-7.05	-3.83	-6.09	-3.28	-1.00			

Table 4-16 continued

(e) Hike in government expenditures / Debt

Birth Year	U.S. Income Class			EU Income Class			Japan Income Class		
	1	2	3	1	2	3	1	2	3
1940	-0.58	0.04	0.08	-1.56	-0.86	-0.38	-1.46	-0.81	-0.36
1960	-1.35	0.19	0.55	-2.27	-1.00	-0.06	-2.24	-0.87	0.18
1980	-3.06	-1.26	0.35	-4.23	-2.79	-0.81	-5.03	-3.40	-1.02
2000	-12.52	-8.55	-5.31	-12.78	-9.58	-4.16	-17.44	-11.93	-5.36
2020	-12.07	-8.49	-4.69	-12.13	-9.59	-4.80	-17.56	-13.65	-6.39
2040	-12.28	-8.76	-4.55	-12.00	-9.45	-4.85	-16.29	-12.39	-6.15
2060	-14.58	-10.75	-6.05	-13.34	-10.59	-5.55	-17.11	-12.81	-6.10
	China Income Class			India Income Class					
	1	2	3	1	2	3			
1940	-3.70	-3.40	-3.23	-0.97	-0.85	-0.80			
1960	-7.55	-5.66	-3.38	-3.29	-2.17	-0.53			
1980	-9.34	-7.51	-5.00	-6.07	-4.68	-2.43			
2000	-16.88	-11.99	-7.81	-11.75	-8.78	-5.68			
2020	-17.80	-12.55	-7.84	-12.43	-8.77	-4.91			
2040	-15.00	-10.23	-5.55	-10.67	-6.95	-3.07			
2060	-15.17	-10.35	-5.54	-9.69	-5.85	-2.15			

Similar welfare changes are observed when the increase in healthcare benefits is limited. Note however that now Chinese and Indian people also experience major welfare gains. The reason is of course that we assumed a higher additional increase in healthcare benefits above the increase in GDP per capita compared to the developed regions in the baseline path. When these additional increases are eliminated all Chinese and Indian cohorts experience much more sizeable benefits than their counterparts in the developed world.

If public expenditures are financed by public debt over the first 20 years of the transition the welfare changes differ between the five regions. As already shown in the unilateral scenario, future cohorts in the U.S. experience sizeable losses in their lifetime resources. Similar effects are observed in NEA since public debt and thus future tax burden is also raised over time. The welfare losses are however much smaller than in the U.S. since the increase in NEA's public debt burden is much more moderate. The exact opposite happens in EMU, China and India. Thanks to reductions in their government debt future cohorts benefit in this scenario while old

and middle-aged cohorts experience small welfare losses due to the initially higher tax burden compared to the baseline.

Again, the welfare effects in the scenario that assumes a multilateral hike in government expenditures are similar to the unilateral reform. When the government's budget is closed by taxes, all cohorts in all five regions experience welfare losses. These losses are even larger when the budget is financed by public debt due to the major increases in debt levels and thus much higher future tax burdens in all regions.

5 Summary of results

This chapter has presented two specific simulation models which are applied to analyze the impact of the demographic transition and the economic consequences of specific policy reforms. Although both models have in common the overlapping generation structure from Auerbach and Kotlikoff (1987) they are quite different in their economic structure. Whereas the closed economy model incorporates various sources of uncertainty, the multi-country model features a deterministic income process but disaggregates the production structure, technology and social security systems.

Despite their structural differences, both models demonstrate that the ongoing demographic transition will increase not only future payroll tax rates but also taxes which finance the federal budget. The reason is that the demand for public goods will decrease more slowly than the current tax base, which is mainly the income of working cohorts. The second model also highlights that the demographic change is not only restricted to developed countries. Also currently emerging economies such as China face a dramatic change in their population structures. Due to their different structure, the models predict a different impact of aging on factor markets. The closed economy model for the German economy predicts a fall in future interest rates due to ageing, since the reduction in the work force dampens effective labor growth more than capital accumulation. Due to the consideration of additional social security systems, however, payroll taxes increase much more in the multi-country model. As a consequence, savings and capital accumulation is dampened much more than labor growth so that world interest rates increase during the transition.

With respect to policy reforms the German model demonstrates that increases in the retirement age under public pension programs are an important policy instrument in order to distribute resources to future cohorts. The simulated increase in the normal retirement age increases output and employment and reduces future payroll and consumption taxes. However, two caveats have to be kept in mind. First, although the impact on economic variables is in the right direction, it is fairly modest. A one year longer work life decreases tax rates by less than one percentage point. Second, policies have to be implemented with care, since incentives such as the proposed higher adjustment factor may induce severe efficiency losses.

During the past years, many countries have implemented policies which reduce the progressivity of their pension systems in order to improve labor market incentives. However, ageing will reduce future benefit levels, so that many countries will experience an increase in old-age poverty similar as in Germany. This development may reverse the current trend and induce reforms which increase the progressivity of pension systems again. Our simulation results indicate that although higher progressivity will reduce economic activity it may still enhance economic efficiency since the insurance gains dominate the implied negative labor market distortions. Of course, this result crucially depends on the assumption about individual risk preferences. But the degree of risk aversion required for this to occur seems to be fairly low.

Of course, the most obvious reform direction in order to fight ageing is a partial or complete switch towards a funded system. It induces the strongest redistribution towards future cohorts at the cost of existing cohorts. It also induces significant increase in retirement ages and reductions in future payroll and consumption taxes. Finally, it may even improve economic efficiency, but this result depends on various economic and political parameters. In our specific set up, privatization reduces economic efficiency in the deterministic income version of the model.

Often in policy analysis the differences between a closed and an open economy are disregarded. Comparing the privatization results in the closed and the open economy model indicate another important warning. While funding induces an increase in aggregate savings this may not lead to a greater capital accumulation in the home country. The unilateral policy experiment in

the multi-country model clearly indicate that reforms in developed countries may only foster economic growth in fast growing Asian countries, although world interest rates decrease significantly. Similarly, the reduction in health care benefits does not accelerate economic growth since it only induces workers to consume more leisure. On the other hand, such a policy may be effective in shifting resources towards future cohorts without hurting existing cohorts too much.

Changes in the level and the financing structure of public expenditures have only a modest impact on the macroeconomy. However, they may dramatically burden future cohorts. Without additional arguments there could be no economic support for them.

Appendix A: Computational appendix for the stochastic model

This section gives an overview over the solution methods used to solve our model numerically. We distinguish between a micro- and a macroeconomic solution method. The former is used to solve the household problem, while the latter serves to compute equilibrium prices and quantities.

A.1 Solving the household problem

In order to compute a solution of the complex household problem, we discretize the continuous elements of our state space \mathcal{Z} . We therefore choose $\hat{\mathcal{A}} = \{a^1, \dots, a^{n_A}\}$, $\hat{\mathcal{P}} = \{ep^1, \dots, ep^{n_P}\}$, and $\hat{\mathcal{E}} = \{\eta^1, \dots, \eta^{n_E}\}$. We use the algorithm described in Tauchen and Hussey (1991) to obtain an approximation to the distribution of η with our set \mathcal{E} and a suitable probability function $\hat{\pi}(\cdot|\cdot, \cdot)$. For all the resulting discrete values of z_j we compute the optimal decision of households from the household optimization problem described above. Since $V_{t+1}(\cdot)$ consequently is also only known in a discrete set of points z_{j+1} , this maximization problem can not be solved analytically. Therefore we have to use the following numerical maximization and interpolation algorithms to compute households optimal decision:

1. Compute household decisions at the last possible age J for all possible z_J . Note that the terminal condition $V_t(z_{J+1}) = 0$, households are not allowed to work anymore and they die for sure in the next period. Hence, they consume all their resources.
2. Find the solution to the household optimization problem for all possible z_j recursively using Powell's algorithm, see Press et al. (2001, 406ff.). Since this algorithm requires a continuous function, we have to interpolate $V_{t+1}(z_{j+1})$. Having computed the data $V_{t+1}(z_{j+1})$ for all $z_{j+1} \in \mathcal{S} \times \mathcal{A} \times \mathcal{P} \times \mathcal{E} \times \mathcal{D} \times \mathcal{R}$ in the last step, we can now find a piecewise polynomial function $sp_{t+1,j+1}$ that satisfies the interpolation conditions

$$sp_{t+1,j+1}(a^k, ep^l) = EV(z_{j+1}) \quad (\text{A-1})$$

for all $k = 1, \dots, n_A$, $l = 1, \dots, n_P$. In this paper we use multidimensional spline interpolation, see Habermann and Kindermann (2007).

A.2 The macroeconomic computational algorithm

The computation method for the macroeconomic model follows the Gauss-Seidel procedure of Auerbach and Kotlikoff (1987). We start with a guess for quantities and government policy. Then we compute prices, optimal household decisions, and value functions. This involves a discretization of the state space which is explained in the previous section. Next we obtain the measure of households and new macroeconomic quantities as well as the social security tax rate and the consumption tax rate that balances government's budgets. This information allows us to update the initial guesses. The procedure is repeated until the initial guesses and the resulting values for quantities, prices and public policy have sufficiently converged.

Appendix B: Computation of the world economy's dynamic equilibrium

To compute the world economy's perfect-foresight general equilibrium transition path, we start with initial guesses of the equilibrium factor and product prices of $r(t)$, $w(t, k)$, $q(j, t)$. We next use these prices to determine consumption demands, $C(j, t, z)$ and supplies of both labor, $L^s(t, k, z)$ and assets, $A(t + 1, z)$ for each year in each region z . In our model, services (*se*), housing (*ho*) as well as the public good g are non-traded goods, i.e. $G_N = \{se, ho, g\}$. Traded consumption goods are the low tech (*lo*) and the high tech (*hi*) good as well as the investment good *in*, i.e. $G_T = \{lo, hi, in\}$. Equilibrium in the traded goods market requires that world output equals world demand

$$Y_W(j, t) = \sum_{z \in W} C(j, t, z) \quad j = lo, hi; \quad (\text{B-1})$$

$$Y_W(j, t) = \sum_{z \in W} [A(t + 1, z) - B(t + 1, z)] - (1 - \delta) \sum_{z \in W} [A(t, z) - B(t, z)] \quad j = in \quad (\text{B-2})$$

where $Y_W(j, t)$ stands for total worldwide production of good j in year t and z references a region in the set $W = \{\text{U.S.}, \text{EMU}, \text{NEA}, \text{China}, \text{India}\}$. Note that world output of the investment good is the difference between the world capital stock in $t + 1$ and t , which, in turn, is the difference between world wealth at $t + 1$ and t .

Factor demands in the non-traded sectors $j \in G_N$ in each region z are determined by the first-order conditions

$$K(j, t, z) = \frac{\alpha(j)q(j, t)}{r(t) + \delta_K} C(j, t, z) \quad (\text{B-3})$$

$$L(j, t, k, z) = \frac{[1 - \alpha(j)]\beta(j, k)q(j, t)}{w(t, k)} C(j, t, z). \quad (\text{B-4})$$

Next, we define the remaining supply of skill-specific labor and capital for traded goods in each

country z and year t , $L_T^s(t, k, z)$ and $A_T(t, z)$, respectively, from

$$L_T^s(t, k, z) = L^s(t, k, z) - \sum_{j \in G_N} L(j, t, k, z) \quad (\text{B-5})$$

$$A_T(t, z) = \mathcal{A}(t, z) - \sum_{j \in G_N} K(j, t, z) - B(t, z). \quad (\text{B-6})$$

World factor markets must satisfy

$$L_W^s(t, k) = \sum_{z \in W} L_T^s(t, k, z) = \sum_{j \in G_T} L_W(j, t, k) = \sum_{j \in G_T} \theta_L(j, t, k) Y_W(j, t) \quad (\text{B-7})$$

$$A_W(t) = \sum_{z \in W} A_T(t, z) = \sum_{j \in G_T} K_W(j, t) = \sum_{j \in G_T} \theta_K(j, t) Y_W(j, t) \quad (\text{B-8})$$

with $\theta_L(j, t, k) = \frac{L_W(j, t, k)}{Y_W(j, t)}$ referencing labor of skill type k per unit of output of good j , in year t , $\theta_K(j, t) = \frac{K_W(j, t)}{Y_W(j, t)}$ referencing capital per unit of output of good j , in year t , and $L_W^s(t, k)$ and $A_W(t)$ referencing, respectively, the world-wide excess supplies of labor and capital available for productive use in the traded sectors given the world-wide demands for these factors to produce non-traded goods. Total demand of labor of skill type k and capital in the traded goods sector j in year t is defined by $L_W(j, t, k) = \sum_{z \in W} L(j, t, k, z)$ and $K_W(j, t) = \sum_{z \in W} K(j, t, z)$. The second equalities in the above two equations (B-7) and (B-8) state that the excess supplies of inputs available to produce traded goods must equal their respective world-wide demands by traded goods producers. Since production technologies for traded goods are identical in all countries we also have

$$\theta_L(j, t, k) = \frac{L(j, t, k, z)}{Y(j, t, z)} \quad \text{and} \quad \theta_K(j, t) = \frac{K(j, t, z)}{Y(j, t, z)} \quad j \in G_T. \quad (\text{B-9})$$

Using (B-9) we can substitute (4-18) into (B-7) and (4-17) into (B-8), providing the following formulas for the capital and labor input-output coefficients in the investment good sector at time

t :

$$\theta_L(in, t, k) = \left[\sum_{j \in G_T} \frac{[1 - \alpha(j)]\beta(j, k)q(j, t)}{[1 - \alpha(in)]\beta(in, k)q(in, t)} Y_W(j, t) \right]^{-1} L_W^s(t, k) \quad (\text{B-10})$$

$$\theta_K(in, t) = \left[\sum_{j=3}^5 \frac{\alpha(j)q(j, t)}{\alpha(in)q(in, t)} Y_W(j, t) \right]^{-1} A_W(t). \quad (\text{B-11})$$

The producer price of the investment good is normalized to unity, i.e. $q(in, t) = 1.0$.

Next, we update our guesses of wages and interest rates by inverting (4-17) and (4-18)

$$r(t) = \frac{\alpha(in)}{\theta_K(in, t)} - \delta_K \quad (\text{B-12})$$

$$w(t, k) = \frac{(1 - \alpha(in))\beta(in, k)}{\theta_L(in, t, k)}. \quad (\text{B-13})$$

In order to derive producer prices $q(j, t)$ for private and public consumption goods we first compute the respective capital and labor input-output coefficients, $\theta_K(j, t)$ and $\theta_L(j, t, k)$. Substituting the two first-order conditions (4-17) and (4-18) into the per-unit demand functions for capital and income-class specific labor we derive, after rearranging,

$$\theta_K(j, t) = \frac{1}{\phi} \frac{\alpha(j)}{r(t) + \delta_K} \left(\frac{r(t) + \delta_K}{\alpha(j)} \right)^{\alpha(j)} \prod_{k=1}^3 \left(\frac{w(t, k)}{[1 - \alpha(j)]\beta(j, k)} \right)^{[1 - \alpha(j)]\beta(j, k)} \quad (\text{B-14})$$

$$\theta_L(j, t, k) = \frac{1}{\phi} \frac{[1 - \alpha(j)]\beta(j, k)}{w(t, k)} \left(\frac{r(t) + \delta_K}{\alpha(j)} \right)^{\alpha(j)} \prod_{k=1}^3 \left(\frac{w(t, k)}{[1 - \alpha(j)]\beta(j, k)} \right)^{[1 - \alpha(j)]\beta(j, k)}. \quad (\text{B-15})$$

The new producer prices for consumption goods $j \in G \setminus in$ are now computed from

$$q(j, t) = [r(t) + \delta_K]\theta_K(j, t) + \sum_{k=1}^3 w(t, k)\theta_L(j, t, k), \quad (\text{B-16})$$

which measures the unit cost of production as the product of input prices and their usage per unit of output.

We next use the factor input-output ratios and the total output of traded goods to allocate production to the different regions. Doing so lets us determine the regional pattern of net exports. Our procedure is to first derive, for each year t and every productivity level k , the excess labor supply $L_T^s(t, k, z)$ in region z available for use in traded goods. We then relate these excess labor supplies to their respective excess labor demands; in so doing, we arrive at three equations in three unknowns, which we use to determine the share of worldwide production of each of the traded goods that occurs in region z .

Equation (B-17) below states that traded-goods excess labor supplies of each skill type in region z (the right-hand side of the equation) equals the sum over the three traded goods of the product of three variables – the input-output coefficient for the type of labor and traded good being considered, the world-wide supply of that traded good, and the share of world-wide supply of that traded good, $\theta_Y(j, z)$, being produced in region z . We have omitted the time index t here for simplicity.

$$\begin{pmatrix} \theta_L(lo, 1)Y_W(lo) & \theta_L(hi, 1)Y_W(hi) & \theta_L(in, 1)Y_W(in) \\ \theta_L(lo, 2)Y_W(lo) & \theta_L(hi, 2)Y_W(hi) & \theta_L(in, 2)Y_W(in) \\ \theta_L(lo, 3)Y_W(lo) & \theta_L(hi, 3)Y_W(hi) & \theta_L(in, 3)Y_W(in) \end{pmatrix} \times \begin{pmatrix} \theta_Y(lo, z) \\ \theta_Y(hi, z) \\ \theta_Y(in, z) \end{pmatrix} = \begin{pmatrix} L_T^s(1, z) \\ L_T^s(2, z) \\ L_T^s(3, z) \end{pmatrix} \quad (\text{B-17})$$

Equation (B-17) represents three equations in the three unknowns $\theta_Y(lo, t, z)$, $\theta_Y(hi, t, z)$, and $\theta_Y(in, t, z)$. Given values of these three variables, we can determine factor inputs in the traded goods sectors from

$$L(j, t, k, z) = \theta_Y(j, t, z)\theta_L(j, t, k)Y_W(j, t) \quad \text{and} \quad (\text{B-18})$$

$$K(j, t, z) = \theta_Y(j, t, z)\theta_K(j, t)Y_W(j, t). \quad j \in G_T \quad (\text{B-19})$$

Our algorithm iterates until the time paths of interest and wage rates converge to a fixed point and supplies for each good equals its demand in every period.

Note that nothing precludes our calculating negative values for the $\theta_Y(j, t, z)$ s. Negative values are, of course, infeasible and indicative of specialization. Recall that our model features three

immobile factors of production (the three labor skill groups), one mobile factor (capital), and three traded goods. These elements suffice for non-specialization and factor-price equalization, but they don't guarantee this outcome. As in the standard 2 by 2 trade model, if relative factor endowments are sufficiently distinct (e.g., one region has very little supply of a particular factor), specialization will arise and factor prices will not be equalized.

In our model, which features exogenous net immigration, we could modify our assumed skill-specific time paths of net immigration to restore factor price equalization were it otherwise to fail to arise. But making such assumptions would be at odds with prevailing immigration policy. An alternative and, actually, seemingly realistic way to restore, where needed, factor price equalization via factor mobility is to take into account the fact that domestic firms are now routinely employing offshore labor (e.g., software engineers, customer support representatives, telephone salespeople) to participate in their domestic productive processes via the internet and telephone. Such foreign employees are, in fact, working with the domestic firm's domestic employees as well as domestic capital in producing domestic output.

To accommodate this economic reality, we set up the following iteration. We set any negative output shares computed via equation (B-17) to zero and compute, using the remaining positive output shares, an updated and, of course, higher labor demand $\tilde{L}(j, t, k, z)$ for each skill level in each region z from equation (B-18). Next, we compute, for each skill group, the difference between total excess labor demand and total excess labor supply, i.e.

$$\Delta L(t, k, z) = \sum_{j \in G_T} \tilde{L}(j, t, k, z) - L_T^s(t, k, z).$$

The excess demand $\Delta L(t, k, z)$ in the traded goods sector of region z is assumed to be acquired via offshore hires. We secure this skill-specific offshore labor by reducing the excess labor supplies for that skill group across the other four regions. The region-specific reductions are allocated in proportion to each of the relevant regions' shares of worldwide labor supply of the skill group in question. Thus, for the four regions \tilde{z} we calculate new excess labor supply as

$$L_T^s(t, k, \tilde{z}) = L_T^s(t, k, \tilde{z}) - \Delta L(t, k, z) \frac{L_T^s(t, k, \tilde{z})}{\sum_{\tilde{z} \in W \setminus z} L_T^s(t, k, \tilde{z})} \quad \text{with } \tilde{z}, \tilde{z} \neq z.$$

Given this resulting modification of excess labor supplies in the traded goods sector in each region, we then recalculate the output shares $\theta_Y(j, t, z)$ from equation (B-17), but now applying the updated $L_T^s(t, k, z)$, and continue iterating in this fashion until all output shares are either positive or zero.

Finally, these offshore hires affect the value of each country's trade balance, which must be adjusted by the respective value of foreign labor imports or exports (labor imports/exports multiplied by the wage rate).

Clearly, other assumptions about the locus of offshore labor hires would result in different region-specific loci of world production. But they would not change the world (five-region) equilibrium transition path. This is just Rybczynski's Theorem (1955) in action; with complete factor mobility, one can't say anything about where products will be produced, although one can say everything about what will be produced and who will get paid what for that production. Of course, the world does not feature complete factor mobility. Our assumption is that the world (our five regions) has only enough factor mobility, including the hiring of labor offshore, to achieve factor-price equalization.

As in the case of solving the full factor-price equalization model, with labor offshoring, our criterion for determining that we have, indeed, iterated to a fixed-point path is to that all goods and product markets, internal and global, as the case may be, clear to many decimal places of accuracy.

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Figure 3-1: Projected population structure

Figure 3-2: Productivity throughout the life cycle

Figure 3-3: Retirement pattern in model and data

*Source: DRV (2010). Own calculations for year 2009.

Figure 4-1: The individual life-cycle

